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RECENT PROGRESS IN THE THEORIES OF MODULAR AND FORMAL INVARIANTS AND IN MODULAR GEOMETRY

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1. Contrast between algebraic and modular invariants. By way of introduction we recall the argument, made in certain texts on invariants, to prove that a linear form $l = ax + by$ has no invariant. For, the vanishing of an invariant $I(a, b)$ of l would imply a property of those forms l for which $I = 0$, not possessed by the forms l for which $I \neq 0$. But all forms l are equivalent since each can be transformed into x. This argument is erroneous since the identically vanishing form l (with $a = b = 0$) cannot be transformed into x. Nor is the conclusion correct. The function $I(a, b)$, defined in the sense of Dirichlet to be unity if $l \equiv 0$ and zero if l is not identically zero, is evidently an invariant of l, since it has the same value for all equivalent forms l.

In the number-theoretic case in which the coefficients of l and of the linear transformation are integers taken modulo p (a prime), the bizarre Dirichlet function $I(a, b)$ employed in the algebraic case is no longer necessary, since it may now be replaced by the polynomial invariant

$$I = (a^{p-1} - 1)(b^{p-1} - 1),$$

with the value unity if $a \equiv b \equiv 0 \pmod{p}$ and the value zero if a and b are not both congruent to zero. Hence I is an invariant of l modulo p. It is called a *modular* invariant of l.

2. Formal invariants and their construction. Let the coefficients a and b of l be independent variables as in the theory of algebraic invariants. But let the coefficients of the linear transformations be integers taken modulo p as in the theory of modular invariants. Invariants arising in this composite case are called formal invariants and were first intro-

duced by Hurwitz.[1] Although a remarkably simple theory of modular invariants has been given,[2] no headway was made towards a theory of formal invariants before the very recent discovery[3] of a simple effective method for their construction. This method will be illustrated for the linear form l and the modulus 2. The real points (i.e., those with integral coördinates) modulo 2 are (1, 0), (0, 1) and (1, 1). The values of l at these points are a, b, $a + b$. Any real linear transformation induces a permutation of these three values since it merely permutes the three real points. Hence any symmetric function of these three values is a formal invariant of l. The elementary symmetric functions reduce modulo 2 to zero, $i = a^2 + ab + b^2$ and $j = ab(a + b)$. We pass to modular invariants by taking a and b to be integers modulo 2. Then $j \equiv 0, i \equiv I + 1$, where I is the invariant in § 1.

In treating similarly the formal invariants of l modulo 5, we would employ the symmetric functions of the fourth powers of our values a, b, $a + b$, and not the values themselves. For, (1, 0), (2, 0), (3, 0), (4, 0) give the same point and yet lead to the values a, $2a$, $3a$, $4a$ of l; we take the fourth power to secure a value uniquely defined by the point. In the case of a quadratic form modulo 5, we need only take the squares of the values.

The method is applicable to invariants of several forms in any number of variables and to semi-invariants, as shown in the paper cited, which gives also a novel method of deriving modular invariants from semi-invariants.

3. Modular plane curves for modulus 2. Let $f(x, y, z)$ be a homogeneous form of degree n with integral coefficients. A point for which the three first partial derivatives of f are zero modulo 2 shall be called a *derived point*. If n is even, it need not be a singular point of the curve, since it need not lie on the curve; the argument in the algebraic case, based on Euler's theorem

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf,$$

does not apply modulo 2 when n is even, since the vanishing of the left member does not require that of f. A non-singular derived point shall be called an *apex* of the curve; its linear polar is indeterminate.

For example, any non-degenerate conic (i.e., having no linear factor modulo 2) can be transformed linearly into $x^2 + yz = 0$. The only derived point is $p = (1, 0, 0)$ and is an apex. Any line through p is tangent to the conic; this is evident for $z = 0$ and follows for $y = kz$ since the elimination of y leads to $x^2 + kz^2 = 0$, with a double root

modulo 2. For the theory[4] of quadratic loci modulo 2 in space of any number of dimensions, see the *Madison Colloquium Lectures*, p. 65.

The next case, $n = 4$, for which apices occur presents other remarkable peculiarities.[5] Whereas in the algebraic case a quartic curve has 28 bitangents in general, one in the case of modulus 2 has at most 7 bitangents and usually exactly 7. The bitangents intersect at derived points and usually all of the derived points are intersections of bitangents.

An interesting example is given by

$$K = x^4 + y^4 + z^4 + x^2y^2 + x^2z^2 + y^2z^2 + xyz\,(x + y + z),$$

an invariant under all real linear transformations. Here real is used in the sense of integral; likewise for the real points $(1, 0, 0), \ldots, (1, 1, 1)$ modulo 2. The bitangents to $K = 0$ are the 7 real lines in the plane and their interesections are the 7 real points, the latter being apices and not singular points.

A quartic curve containing all seven real points and having no linear factor modulo 2 can be transformed into

$$x^3y + x^2y^2 + xz^3 + x^2z^2 + y^3z + yz^3 = 0.$$

It has no singular point and has the 7 apices $(1, z^3, z)$, where $z^7 + z^3 + 1 = 0$. Its 7 bitangents are $x = (b^3 + 1)\,y + bz$, where $b^7 + b + 1 = 0$; they intersect at apices. Each apex is on three bitangents, while three apices are on each bitangent. The configuration of the apices and bitangents, here all imaginary, is entirely similar to that for K, composed of real elements.

The classification of quartic curves is similar to that next illustrated for the simpler case of cubic[6] curves modulo 2.

A cubic curve containing all seven real points is of the form

$$a\,(x^2y + xy^2) + b\,(x^2z + xz^2) + c\,(y^2z + yz^2) = 0.$$

If not zero identically, it can be transformed into $x^2y + xy^2 = 0$. A cubic curve containing just two real points can be transformed into one containing $(1, 0, 0)$ and $(0, 1, 0)$; the transformations leaving the latter fixed or permuting them are available for the specialization of the parameters in the coefficients of the cubic. In this way we find the 21 types of non-equivalent cubics, including degenerate curves, and see that they are completely characterized by the number of real points, real inflexion points, real and imaginary singular points,—geometrical invariants easily expressed by modular invariants.

For the determination of the inflexion points of the cubic $n\,(x_1, x_2, x_3) = 0$ modulo 2, the Hessian of n is not available, being identically zero modulo 2. In its place we may employ the function

$$C = \xi_1\xi_2\xi_3 + \Sigma\,\xi_i\eta_i^2 + k\Sigma x_1\xi_1\eta_1 + k\Sigma x_1\eta_2\eta_3 + k^2\Sigma x_1x_2\eta_3 + k^3x_1x_2x_3,$$

in which $n = v + kx_1x_2x_3$, while the quotients

$$\xi_i = \tfrac{1}{2}\frac{\partial^2 v}{\partial x_i^2},\ \eta_1 = \tfrac{1}{2}\frac{\partial^2 v}{\partial x_2\partial x_3},\ \cdots,\ \eta_3 = \tfrac{1}{2}\frac{\partial^2 v}{\partial x_1\partial x_2}$$

have integral coefficients. The points of inflexion of $n = 0$ are its intersections with $C = 0$. Although C is not a covariant, it forms with n a covariant pencil, since C is transformed into a linear function of n and C.

[1] Hurwitz, *Arch. Math., Leipzig*, ser. 3, 5, 25, (1903).

[2] Dickson, *Madison Colloquium Lectures*, American Mathematical Society, (1914).

[3] Dickson, *Trans. Amer. Math. Soc.*, 15, 497, (1914).

[4] An advance in the theory of seminvariant leaders of covariants of quadratic forms has been made recently by the writer, *Bull. Amer. Math. Soc.*, January, 1915.

[5] Dickson, *Trans. Amer. Math. Soc.*, April, 1915.

[6] MS. offered Aug. 4, 1914 to *Amer. J. Math.* To appear April, 1915.

THE SYNTHESIS OF TRIAD SYSTEMS Δ_t in t ELEMENTS, IN PARTICULAR FOR t = 31

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Purely theoretical interest first led to the study of triad systems Δ_t in t elements; systems of threes or triads, that is, in which every possible pair of elements is found in some one triad, but only one. Their relation to other objects of research in algebra and geometry began to appear when Noether (1879) pointed out the peculiar nature of a resolvent equation of the seventh degree which had been found by Betti, Hermite, and Kronecker in discussing the transformation of the seventh order of elliptic functions. This modular equation has roots related in triads like the Δ_7. Hesse had shown earlier, in plane curves of the third order, that the nine inflexion points lie by threes on twelve lines, thus exemplifying a Δ_9. Noether succeeded also in connecting the Δ_7 with the important sets of double tangents to the plane quartic curve which Aronhold had introduced under the name of *Siebenersysteme*. With those important applications in hand, and ten or twelve known Δ_{15}'s to serve as further data, mathematicians took up with renewed interest the question whether there are actual triad systems for every suitable member t of elements, i.e., for $t = 13, 15, 19, 21, 25, 27$, etc.; or precisely, $t = 6k + 1$ or $6k + 3$.

Although Reiss in 1859 had answered this question in the affirmative, constructing one system for every value of t, his work was overlooked until the same question was settled independently in 1893 by E. H. Moore. Moore's methods, based on more penetrating analysis than Reiss's, established at least two distinct sorts of systems for every t above 13, and led him to forecast definitely that the number of such systems would be found a rapidly increasing function of the number of elements, t. As to the sole doubtful number, $t = 13$, Zulauf, a pupil of Netto, found that there are two different systems, and others soon proved that there are no more than two.

After the lapse of ten years or more, Miss L. D. Cummings has now shown conclusively (1914) the existence of at least 24 distinct triad systems in 15 elements. These 24 include all that had been found before and as many more new systems, with their differences now for the first time rigorously demonstrated. All but one (viz., Heffter's) of these 24 exhibit what I call odd-and-even structure. The odd part are the elements appearing in seven triads that constitute an included triad system Δ_7, which may be termed the head in its Δ_{15}. Heffter's Δ_{15} is at present the only headless system in 15 elements whose description has been published.

Since the appearance of Miss Cummings' dissertation, I have applied a new method for constructing all possible Δ_{15}'s which can be transformed into themselves by any substitution among their elements,—all whose group is above the identity. By this means I find a considerable additional number of systems, all headless. These new Δ_{15}'s I now employ in attacking the question, how many distinct systems Δ_{31} are there in 31 elements? If there were but few, then it would be desirable to compile a complete census of them as further substratum for a general theory. But what I find is that even the restricted class selected for this study are far too numerous for detailed exhibition, their number being greater than 10^{13}. This result is attained through a new theorem, whose generality is significant of further possibilities.

The theorem, specialized for application, is this. *If among the triads of a system* Δ_{31} *there occur two complete systems* Δ_{15} *and* Δ'_{15}, *then there is a* Δ_7 *whose seven triads, and no other triads or elements, are common to* Δ_{15} *and* Δ'_{15}. Conversely, *if a* Δ_{31} *contains a headless* Δ_{15}, *it can contain no other triad system* Δ'_{15}; nor indeed any other larger than a Δ_7, and even such a Δ_7 will have one triad from the Δ_{15}.

Odd-and-even structure in a Δ_{31} consists in this: its 155 triads include 35 that form a Δ_{15} in 15 elements, and of the remaining 16 elements two are found in each of the other 120 triads, along with one element from

the 15. Thus every triad has an odd number, 3 or 1, of the *odd set* of 15, and an even number, 0 or 2, of the *even set* of 16. Any one of the odd set is found therefore in triads with 8 pairs from the even set, and these pairs can be arranged in 15 columns, an 8 by 15 array. Every odd element is found also with 7 pairs from the odd set. This leads to the tabulation of 15 columns of 7 pairs each, ranged above the columns of the 8 by 15 array. Every column is marked then by one odd element above it; the upper partial columns exhibit the head, or Δ_{15}.

Head and array form a convenient mode for constructing Δ_{31}'s that are to have odd-and-even structure. If the head, the Δ_{15}, is itself headless, this tabulation is unique for that Δ_{31}. *I study here exclusively these odd-and-even Δ_{31}'s whose head is a headless Δ_{15}.* Given any one such Δ_{31}, tabulated, many others can be obtained by shifting the columns of its 8 x 15 array while the head is kept stationary. To apply this method and to count the distinct Δ_{31}'s that will be produced, one must know the groups G_d and $G_{d'}$, belonging to the head and to array respectively.

The number of resulting Δ_{31}'s is certainly not less than 15! divided by the product, $d\ d'$, of the orders of the groups belonging to the head and to the array respectively. These orders are small, whence the resulting Δ_{31}'s are very many.

Incidentally, if d and d' are relative primes, the resulting Δ_{31}'s must be of the peculiar kind having no automorphic substitutions; i.e., their group is reduced to the identity. Such cases occur, e.g., with $d = 2$ and $d' = 3$. Full details are to appear in the *Transactions of the American Mathematical Society* for January, 1915.

THE Φ-SUBGROUP OF A GROUP OF FINITE ORDER

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Presented to the Academy, November 19, 1914

A set of λ operators $s_1, s_2, .., s_\lambda$ of a finite group G is called a set of generators of G provided there is no subgroup in G which includes each of these operators. When these operators satisfy the additional condition that G can be generated by no $\lambda - 1$ of them the set is said to be a *set of independent generators of G.* Those operators of G which can appear in none of its possible sets of independent generators constitute a characteristic subgroup, which was called by G. Frattini *the ϕ-subgroup of G.* See *Rend. Acc. Lincei*, ser. 4, 1, 281 (1885).

Among the results of the present paper which are supposed to be new are the following: The number of the different operators in each of the possible sets of independent generators of a group whose order is a power of a prime number is the same,—that is, if the order of a group is a power of a prime number, the number of its independent generators is an invariant of the group. The ϕ-subgroup of every direct product is the direct product of the ϕ-subgroups of the factors of this direct product. In every group whose order is a power of a prime number the ϕ-subgroup includes the commutator subgroup of the group. By constructing the ϕ-subgroup of a group and of its successive ϕ-subgroups we can always arrive at the identity.

The ϕ-subgroup of a Sylow subgroup of the symmetric group of degree n is the commutator subgroup of this Sylow subgroup. In particular, the number of the different operators in a set of independent generators of the Sylow subgroup of order p^m of the symmetric group of degree p^n, p being any prime number, is n. In particular, the subgroup of order p^{p+1} which is contained in the symmetric group of degree p^2 has exactly 2 independent generators irrespective of the prime number represented by p. No ϕ-subgroup can contain a complete set of conjugate subgroups, or a complete set of conjugate operators, involving more than one subgroup or more than one operator, when this complete set of conjugates is also a complete set of conjugates under the entire group. An important special case of this theorem is that every ϕ-subgroup whose order is not a power of a prime number is the direct product of its Sylow subgroups. This special case was noted by G. Frattini in the article to which reference has been given.

With respect to abelian groups a set of λ independent generators is commonly defined so that the group generated by every $\lambda - 1$ of these generators has only the identity in common with the group generated by the remaining operator. For an abelian group whose order is a power of a prime number the number of the different operators in a possible set of independent generators is the same under both of the given definitions of a set of independent generators. The fact that the number of independent generators of a group is not always an invariant of the group becomes evident if we observe that when we generate the symmetric group of degree n by transpositions there will always be $n - 1$ independent generators. On the other hand, this symmetric group can also be generated by a cyclic substitution of degree $n - 1$ and a transposition involving the remaining letter. Complete proofs of these results are contained in a paper having the same heading, which has been offered for publication in the *Transactions of the American Mathematical Society*.

ON THE RADIAL VELOCITIES OF NEBULAE

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The discovery, early in 1910, that the velocities of the stars are functions of their spectral types—the stellar velocities increase rapidly, on the average, as we pass from the blue stars through the yellow stars and on to the red stars—and the prevailing opinion that the stars have been evolved from nebulae, gave special interest to the search for possible relationships existing between nebular and stellar velocities. The nebular velocities then available were the velocities of approach and recession (radial velocities) of 13 planetary nebulae and of the Orion nebula, as observed by Keeler in 1890–1891 with a visual spectroscope attached to the 36-inch refracting telescope. The average radial velocity of the 14 nebulae, as observed by Keeler, was 25 km. per second; or omitting the Orion nebula, whose velocity is close to zero, 26½ km. per second for the 13 planetaries on his list. The average radial velocity of the helium stars, very generally supposed to be the stars most recently formed from nebulae, was found to be only 6 km. per second. The average nebular velocity was four times as great. Were Keeler's results, only 14 in number, too few to furnish a safe statistical basis of comparison? The basis was at least too small to support a superstructure of any pretensions to reliability. A knowledge of a greatly increased number of nebular velocities was strongly demanded.

During the past three years, at the Lick Observatory and at the associated D. O. Mills Observatory, Santiago, Chile, more than 200 measures of nebular radial velocities have been secured by spectrographic methods. These relate to 61 nebulae, but as 7 of these objects have been observed only once each, this note is limited to the velocities of 54 nebulae.

The distribution of the velocities, as to magnitude, tabulated below is remarkable. No distinction between velocities of approach and velocities of recession is made in this table.

Velocities, km./sec.	0–10	10–20	20–40	40–60	60–80	80–up
Number of nebulae	11	6	17	11	2	7
Average velocity	5	16	28	50	77	132

Total number of nebulae 54. Grand average velocity 42 km./sec.

It is now well known that the radial velocities of four-fifths of the so-called helium stars are under 10 km. per second, and that the average

for all helium stars is about 6 km. per second. Of the observed nebular velocities only one-fifth are under 10 km. per second, and the average for the 54 observed objects is 42 km. per second, or 7 times that of the helium stars. The velocities of the helium stars follow pretty closely the so-called 'probability curve'—a great number of small velocities and very few large velocities. The nebular velocities do not follow the probability curve at all.

If we omit velocities greater than 60 km. per second, as abnormal, we find the *average radial velocity of 45 nebulae to be 26.4 km. per second*, which is still 4 times the average for all the helium stars.

Omitting 12 extended and ring nebulae we find that *the average radial velocity of 42 planetary nebulae is 46.1 km. per second.*

The velocities of the extended and ring nebulae seem to be habitually low, but the number of these included in the program thus far is too small to serve as a safe basis for drawing conclusions. The nebulae N. G. C. 6644 (α = 18h 26m, δ = $-25.2°$) and N. G. C.$_2$ 4732 (α = 18h 28m, δ = $-22.7°$), which are less than 3° apart, have velocities 202 km. per second recession, and 141 km. per second approach, respectively—a relative radial motion of 343 km. per second.

In general, assuming that the motions of celestial bodies are substantially at random in direction and speed, the average radial velocity of a considerable number of objects is exactly one-half of the average space velocity of the objects. We shall probably not be far wrong in assuming that the average radial velocity of 42 km. per second for the 54 nebulae means that the average space velocity of the same objects is approximately 84 km. per second.

All of the above results have been freed from the effects of the sun's motion as determined from spectrographic observations of the stars.

The algebraic mean of the 54 observed velocities, after correcting for the effects of the sun's motion, is -2 km. per second. There is thus no indication of a systematic tendency of the planetary nebulae for motions of approach or of recession.

These observations refer exclusively to nebulae whose spectra are composed chiefly of bright lines, and there are no spiral nebulae on this list.

With reference to stellar motions, the planetary nebulae are rapid travellers. Does this fact throw any light upon their origin or their possible relation to stars? A few of the stars are known to be travelling rapidly. These rapid travellers are found mostly amongst the yellow and red stars, which are usually held to be furthest removed from the nebular condition. Perhaps the best explanation of the so-called novae,

or new stars—stars which suddenly flash up where previously no stars, or at the best, very faint stars, were known to exist—is that they are due to the rushing of dark or relatively faint stars through invisible nebulae or resisting media. The resulting collisions, in effect a bombardment of the stellar surfaces, seem to generate sufficient heat to convert the surface strata of the stars into incandescent gases and vapors. Spectrographic observations have shown for all recent novae that in the course of a few weeks or months the spectra of the novae are converted into the nebular type. Later the nebular spectra disappear and a certain type of stellar spectrum takes their place. These changes occur rapidly, perhaps because the bombardment effects have been but skin deep.

That planetary nebulae may have resulted in some such way from the collisions of stars and the resisting media seems quite possible. Exactly those stars which are travelling with very high speeds would have the greatest chance to encounter resisting media: and further, the disturbances would be deeper and more permanent the higher the speed of collision. Conversely, if this speculation has a basis in fact, that is, if the planetary nebulae have been formed in this manner, the prevailing high velocities would find a natural explanation.

In securing the difficult long-exposure observations upon which this note is based, I have been greatly assisted by several members of our staff, especially by J. H. Moore, E. S. Haynes, and P. W. Merrill.

PRELIMINARY NOTE ON NEBULAR PROPER MOTIONS

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The great number of nebulae in the sky, their well-known grouping in regions distant from the Milky Way, the spiral structure so frequently occurring, and the question as to the place of these bodies in the evolution of the sidereal universe, all present a series of problems as yet only partially solved.

A knowledge of the proper motions or of any rotational movements which these bodies may have would be of very great value in investigations as to the size and distance of the nebulae, and therefore as to their place in the structure of the visible universe. Owing to the hazy and diffuse character of the great majority of nebulae when observed visually, the visual observations of the past, just as they have proved nearly valueless in giving an adequate conception of the form and structure

of these remarkable objects, so they are also inadequate as records of position for the determination of proper motions and rotations. Trustworthy results can be secured only by the aid of photography.

Sixteen years have now elapsed since Keeler inaugurated his program of nebular photography with the Crossley Reflector of the Lick Observatory; and Keeler's plates, in connection with recent photographs secured with the same instrument, afford a large mass of material for the investigation of nebular motions. Some of the diffuse objects of the Keeler program do not admit of precise measurement; some also of the old plates are rather poor; but when the program of repetition, commenced early in 1914, shall be completed, in 1915, sets of photographs, early and recent, with an average interval of time of about thirteen years, will be available for the determination of the proper motions or internal movements and changes of between 80 and 100 nebulae.

This program is now about one-third complete, as regards the taking of the modern plates, and about one-fourth of the measurements have been made. The work to date comprises most of the larger spirals, such diffuse nebulosities as the Orion and Network nebulae, several planetaries, and several very faint small nebulae

As the early and late plates of this investigation have been secured with the same instrument, the methods of measurement have been made entirely differential. Five or six symmetrically-placed small stars are selected as a common reference system for all the plates of a given nebula, and the nucleus and numerous knots or condensations are measured in each nebula as available.

Detailed results will be published upon the completion of the program, but from the observations already made we may draw the following general conclusions, which will probably not be seriously modified by the inclusion of more objects:

1. In this average interval of thirteen years it has not been possible to detect any evidence of internal movement, rotatory or otherwise, in the nebulae measured.

2. The derived proper motions are so small that they may well be ascribed very largely to the difficulty and uncertainty of measuring these difficult objects. The largest proper motions found are for the planetary nebulae N. G. C. 6905 and 7009: 0.056″ and 0.054″ per year, respectively. Those for the other objects measured will average two or three hundredths of a second of arc per year. As the plates are exposed from two to four hours, and as the nebular condensations are much less sharp than star images, the errors of measurement are manifestly much larger than they would be in the case of short-exposure stellar images.

So far as the program has gone, therefore, all the evidence points to a great distance for these objects. As the spirals are undoubtedly in revolution—any other explanation of the spiral form seems impossible—the failure to find any evidences of rotation would indicate that they must be of enormous actual size, and at enormous distances from us. At a distance of one-thousand light-years, supposing mass conditions to be the same as in our solar system, a nebular structure one minute of arc in apparent distance from its nucleus, or 60 times the distance of Neptune from the Sun, would have a yearly angular movement of only 53″, or a maximum yearly displacement in a given component with reference to the nucleus of only 0.015″. A much greater time interval will probably be necessary before nebular rotations can be definitely established by measures of position, unless a nebula abnormally close to us be found.

Wolf and Slipher have reported the observation of rotation in the spiral nebulae Messier 81 and N. G. C. 4594, respectively, by means of the velocity-displacements of lines in the spectra of these bodies. The spectrographic method, here as elsewhere, is independent of the distance or absolute size of the object.

DISCOVERY OF THE NINTH SATELLITE OF JUPITER

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The Ninth Satellite of Jupiter was discovered on photographs taken with the Crossley Reflector of the Lick Observatory, on July 21 and 22, 1914. The photographs were of two and one-half hours exposure, and were taken to secure observations of the Eighth Satellite. The new satellite is fainter than the Eighth, and is estimated as nineteenth magnitude. The identity of the object as a satellite was established from the observations of July 22, 27, and 31 by Leuschner's *Method of Direct Solution of Orbits of Disturbed Bodies*. The motion is retrograde, and the first estimate of the period places it at about three years. The remaining elements of the orbit resemble those of the Eighth Satellite. Additional observations of the satellite have been secured in August and September. A detailed account of the discovery and observations, and of the investigation of the orbit, will be published in a *Lick Observatory Bulletin*.

SPHERICAL ABERRATION IN ASTRONOMICAL OBJECTIVES DUE TO CHANGES OF TEMPERATURE

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Presented to the Academy, November 5, 1914

In September of this year the 30-inch objective for the Thaw Photographic Refractor was installed at the Allegheny Observatory. The objective was made by the John A. Brashear Company of this city. Before it was removed from the optical shop it was tested in a number of ways by the makers and by me. These tests implied an almost perfect figure, and also indicated that the material in the objective is of unusual excellence.

When the objective was first tested at the telescope on actual images of stars, a considerable amount of positive spherical aberration was found to be present; that is, the focal lengths of the outer zones were found to be shorter than for the inner and intermediate zones. The tests were repeated on as many different occasions as possible, with the result that while positive aberration was found to be always present, it varied in amount from night to night and even in the course of a single night. A study of the data thus obtained indicates that the aberration arises from effects of temperature and that it is more pronounced when the temperature is falling rapidly.

To remove this aberration the following expedients were employed. During the day-time, when the telescope is not in use, the objective is enclosed by a metallic cover. With the tube of the telescope this cover acts as a kind of temperature-case, throughout which the temperature is roughly uniform. The observer removes this cover about one hour before beginning to observe in the evening, and at the same time he starts a small electric fan mounted near the upper end of the tube. This fan blows a continuous current of air into the tube through a large vent just below the rear surface of the objective; two similarly placed vents permit the air to escape from the tube. In this way the warmer air that has accumulated during the day is expelled and the rear surface of the objective cools as rapidly as the front. The upper (flint) lens of the objective is separated from the lower (crown) lens by a little less than half an inch. The iron cell in which the two are mounted has been pierced by six holes half an inch in diameter; these permit the air to gain access to the space between the two components.

At the close of the night the observer covers the objective, closes the three vents and inserts cork stoppers in the six lateral holes.

The fan is on the same circuit with the electric lights that illuminate the setting circles of the telescope. As a consequence the fan is in operation at frequent intervals throughout the night. As the lights must be turned off whenever an exposure is being made, the fan is out of operation and cannot cause the tube to vibrate at any time that this could be harmful.

These devices have proved successful in removing the aberrations due to temperature. The Hartmann test has since been applied on a number of occasions and has invariably indicated an excellent figure for the objective.

It is not to be supposed that the Thaw objective is peculiar in its behavior with respect to changes in temperature. An examination of the literature on this subject with regard to other objectives reveals the fact that some of them, at least, are similarly affected and exhibit aberrations that change from time to time, precisely as the Thaw objective did before the application of the ventilating devices. It therefore appears to me that the installation of similar arrangements in the case of other telescopes might bring about a considerable improvement in their performance.

The details of this investigation are to appear in Volume 4 of the Publications of this Observatory.

THE RELATIONS BETWEEN THE PROPER MOTIONS AND THE RADIAL VELOCITIES OF THE STARS OF THE SPECTRAL TYPES F, G, K, AND M

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Among the stars observed for radial velocity with the 60-inch reflector of the Mount Wilson Observatory, there are many of the spectral types *F*, *G*, *K*, and *M* which have either very large or very small astronomical proper motions. The two classes of stars show some very marked differences which indicate a close relationship between proper motion and radial velocity. It is the aim of the present note to bring out this relationship as clearly as possible.

For this purpose it is necessary to supplement the Mount Wilson data for the stars of extreme proper motion with similar data for the rest of the stars. There is, of course, no better source for this purpose than the catalogue of radial velocities published recently by the Lick Observa-

tory.[1] The Bonn observations by Küstner came to hand too late for any extended use. The classification used is that of the Harvard College Observatory, and the spectra given in Campbell's catalogue are, with a very few exceptions, those of the Harvard Revised Photometry.[2] The Mount Wilson stars have been classified by the method described in the paper by Adams and Kohlschütter.[3] Occasionally large differences have been found from the estimates of the Harvard observers for the same stars.

The observed radial motions were first reduced to velocities (ρ) relative to the center of gravity of the stellar system by applying the component of the sun's motion through space with reversed sign; that is, the correction $V \cos\theta$, where V is the sun's velocity, and θ the angular distance between star and apex. For the apex we adopted the position α (1900) = 18h, 0m; $\delta = +31°$, and for V the value 20.0 km. per second. The values of ρ thus obtained were grouped: (1) according to amount of proper motion; (2) according to distance λ from the nearest true vertex of the two star-streams. The positions of these vertices, according to the determinations of Kapteyn, Eddington, and others are:

Stream I, 18h 12m $-12°$; Stream II, 6h 12m $+12°$

The results are summarized in Table I. The letter μ denotes the total proper motion, and W and C refer to the radial velocity determinations by the Mount Wilson observers and by Campbell, respectively. Average values are indicated by dashes over the letters, and figures in parentheses denote the number of stars on which each average is based. The two columns under λ are, as a rule, limited by values of λ 60° to 90° ($\bar{\rho}_1$), and 0° to 49° ($\bar{\rho}_2$). A third group between the narrow limits 50° to 59° was formed, but is omitted from the table. In a few cases where the number of stars was small this middle group was used to increase the weight of the extreme groups. Six stars of abnormal velocity have been excluded out of a total of 1106 stars. These are:

Lal. 15290 −249.4	A.Oe 20452 −163.0	Boss 130 − 80.7
Boss 315 + 62.8	Boss 1511 +163.0	A.Oe 14320 +299.4

An inspection of Table I shows clearly: first, that the average velocity of the stars near the vertices is considerably greater than that of the stars far away from these points; second, that the average value of ρ increases rapidly with the proper motion.

The first of these conclusions is indicated most concisely by the values of the ratio $\bar{\rho}_2/\bar{\rho}_1$ in the last column. It is a necessary consequence of the existence of two star streams with opposite motions. Thus, it is

evident that if all of the stars moved accurately in the direction of these streams the radial velocity would be zero at those points where the line

TABLE I

AVERAGE VALUES OF THE RADIAL VELOCITY ρ IN KM.

TYPE	PROPER MOTION μ	AUTHORITY	λ 60° to 90° $\bar{\lambda}$	$\bar{\rho}_1$		λ 0° to 49° $\bar{\lambda}$	$\bar{\rho}_2$		TOTAL $\bar{\lambda}$	$\bar{\rho}$		$\bar{\rho}_2/\bar{\rho}_1$ Obs.	Theory
F	0.000″ to 0.029″	W+C	79°	8.9	(25)	32°	13.6	(22)	56°	10.6	(54)	1.53	1.63
	0.030 0.069	C	71	8.5	(9)	29	12.5	(11)	48	10.15	(22)	1.47	1.56
	0.070 0.149	C	78	12.3	(17)	34	20.5	(17)	53	14.65	(38)	1.67	1.58
	0.150 0.249	C	72	11.5	(22)	32	18.9	(12)	58	14.3	(40)	1 64	1.55
	0.250 0.499	C	74	16.7	(24)	33	21.1	(8)	62	16.2	(38)	1.26	1.54
	≧0.500	W+C	73	18.05	(30)	43	34.9	(18)	62	24.4	(48)	1.93	1.39
G	0.000 to 0.026	W+C	73	6.9	(37)	32	12.8	(32)	54	9.6	(80)	1.86	1.55
	0.027 0.049	C	76	9.6	(10)	37	12.8	(13)	54	11.8	(28)	1.33	1.53
	0.050 0.099	C	74	9.5	(18)	27	12.3	(7)	61	10.3	(25)	1.30	1.63
	0.100 0.499	C	76	14.95	(13)	44	23.6	(11)	60	19.0	(28)	1.58	1.41
	≧0.500	W+C	76	22.9	(51)	37	45.6	(31)	60	32.1	(91)	1.99	1.51
K	0.000 to 0.025	W+C	76	10.9	(47)	36	12.3	(39)	57	11.7	(91)	1.13	1.52
	0.026 0.039	C	73	12.15	(37)	37	11.9	(24)	58	11.7	(68)	0.98	1.47
	0.040 0.059	C	75	14.9	(21)	38	21.3	(21)	56	17.9	(50)	1.43	1.49
	0.060 0.079	C	75	10.7	(24)	36	15.1	(16)	58	12.3	(41)	1.41	1.50
	0.080 0.099	C	74	15.7	(26)	34	14.0	(10)	62	15.2	(36)	0.89	1.52
	0.100 0.119	C	76	20.7	(12)	32	22.6	(15)	52	23.4	(31)	1.09	1.58
	0.120 0.149	C	73	16.5	(24)	39	23.8	(10)	62	19.0	(36)	1.44	1.43
	0.150 0.199	C	74	15.6	(24)	39	24.5	(10)	62	18.2	(34)	1.57	1.45
	0.200 0.299	C	75	16.5	(18)	32	24.3	(13)	56	20.4	(37)	1.47	1.56
	0.300 0.599	C	73	33.3	(12)	33	47.05	(7)	58	38.35	(19)	1.41	1.52
	≧0.600	W+C	74	23.5	(25)	34	23.0	(28)	53	22.5	(58)	0.98	1.52
M	0.000 to 0.029	W+C	76	12.9	(18)	36	18.1	(15)	58	14.4	(40)	1.40	1.52
	0.030 0.089	C	79	15.0	(19)	36	21.0	(11)	62	17.0	(34)	1.40	1.55
	0.090 0.499	C	77	17.9	(14)	35	28.9	(7)	62	20.7	(24)	1.61	1.56
	≧0.500	W	71	53.2	(4)				71	53.2	(4)		

of sight is at right angles to the stream motion; that is, where $\lambda = 90°$. On the other hand the motion would be altogether radial at the vertices themselves; that is, ρ should be large for $\lambda = 0°$. Since the motions of the stars do not in general coincide with the stream motion, but only show a preference for directions differing but little from it, the contrast in the amount of the radial motion at $\lambda = 90°$ and $\lambda = 0°$ will be less marked. This amount can, of course, be calculated as soon as we have derived complete elements for the two streams from a consideration of the proper motions. As it is, different theories give slightly different values,[4] and eventually these differences may serve to determine the theory to be

preferred. If we use that developed by Kapteyn[5] we obtain the comparison shown in Table I. Except for some of the *K*-type stars the agreement of the values is very satisfactory. The theory, of course, requires that ρ be proportional to $1 + \cos^2 \lambda$.

The values of $\bar{\rho}_2/\bar{\rho}_1$ are no smaller for the stars having the very smallest proper motions than for the other stars in the list. This proves that the two star streams extend to the greatest distances for which we have means of judging. This fact has been doubted by Eddington[6] who proposed an attractive explanation of the behavior of the helium stars, which show hardly a trace of the second stream, on the supposition that the greater part of them are beyond the region where we must admit the existence of two streams. Several objections have already been raised against this hypothesis,[7] and we believe that Eddington has abandoned it. It is interesting, however, to find in the radial motions of these stars so strong a proof of the great extent of the star streams.

The low value of $\bar{\rho}_2/\bar{\rho}_1$ for some of the *K* stars, particularly for some of those of very small proper motion, seems difficult of explanation. A preliminary investigation of the proper motions for the stars having values between 0.026″ and 0.039″ indicates that members of the second stream are not wanting. Evidently the anomally will require much further study.

The second conclusion indicated by Table I, that ρ increases with increase in proper motion, may be explained in any one of at least three ways. We may assume that:

a. The real velocity of the stars decreases with the distance.

b. The more luminous stars move more slowly than the fainter ones.

c. The distribution of the velocities of the stars is not in accordance with Maxwell's law, the large velocities being in excess.

The application of the first two of these explanations is evident. In the consecutive groups for each spectral type the average magnitude is roughly the same. Hence, with the average proper motions increasing regularly from one group to another, the average distances must decrease, and the luminosities become less.

The possibility of the third explanation, *c*, may seem somewhat less evident. Its consideration, however, is essential since it has already been shown by Schwarzschild[8] that the distribution of velocities, using values relative to the sun, cannot agree with Maxwell's law, and that the larger values must be in excess. The radial velocities used here afford the data for the derivation of the velocity-law relative to the center of the stellar system. Such a derivation we have actually carried out in the

case of the K stars, and we hope in the course of a more extended discussion to describe it in detail, as well as to obtain similar expressions for the stars of other types. A very brief summary of the method used is as follows:

1. All of the stars between the limits $\lambda = 60°$ and $\lambda = 90°$ were selected from Campbell's catalogue and arranged according to amount of radial velocity. This selection was made in order to eliminate the effect of stream motion so far as possible. The application of Maxwell's law to this material at once showed a large preponderance of high velocities. Thus there are 17 stars with velocities above 40 km. per second where Maxwell's law requires less than one-third of this number.

2. A satisfactory expression for representing the observed distribution of velocities was found in the sum of two Maxwell equations with different moduli. A peculiar feature of this result is that if all of the stars are used, and not alone those on which the stream motion has little influence, the exponential constants remain essentially unchanged.

3. With the aid of this expression the relationship was determined between the average radial velocity and the component of the linear velocity at right angles to the line of sight.

4. The stars were then divided into groups, each within a narrow range of proper motion, so that they may all be regarded as at the same distance. The components of the linear velocities at right angles to the line of sight were then computed by the aid of the parallaxes given in *Groningen Publications* No. 8. A factor was applied to the parallaxes such as to make the total average linear velocity for all of the stars equal to the total average radial velocity ρ. Table II shows the final results for ρ.

It appears from these results, therefore, that our assumption c is quite adequate to explain the variation of radial velocity with proper motion. There remains, however, the possible effect of a or b, which we may designate briefly as the effect of distance or of absolute magnitude. With more extensive data it might perhaps be possible to separate these two effects. As it is we shall suppose the distance effect to be negligible and try to determine the absolute magnitude effect.

For stars of approximately the same distance we shall assume that $\rho = a + b\,M$, where M is the absolute magnitude derived from the formula,[9] $M = m + 5 + 5 \log \pi$. In general we have kept the same groups used in Table I, the variation in distance not being excessive, but in a few cases consecutive groups have been combined. These groups were then further divided according to apparent magnitudes, an attempt being made to keep the numbers of the stars in the extreme subdivisions about equal. The values of the parallax π were then taken from *Gronin-*

gen Publications No. 8, and the absolute magnitudes were computed. From the resulting equations the values of b were determined by least squares.

TABLE II

COMPUTATION OF THE RADIAL VELOCITY AS A FUNCTION OF THE PROPER MOTION

PROPER MOTION μ	NO. STARS CAMPBELL	PARALLAX π GRON. NO. 8	$\bar{\rho}$ OBS.	$\bar{\rho}$ COMP.	DIFFERENCE
seconds (″)		*seconds* (″)	*km.*	*km.*	*km.*
0.000 to 0.025	27	0.0064	10.9[1]	12.1	−1.2
0.026 0.039	37	0.0112	12.15	12.5	−0.35
0.040 0.059	21	0.0148	14.9	12.9	+2.0
0.060 0.079	24	0.0185	10.7	13.3	−2.6
0.080 0.099	26	0.0218	15.7	13.7	+2.0
0.100 0.119	12	0.0248	20.7	14.3	+6.4
0.120 0.149	24	0.0283	16.5	14.8	+1.7
0.150 0.199	24	0.0332	15.6	15.9	−0.3
0.200 0.299	18	0.0412	16.5	17.7	−1.2
≦0.300	19	0.108	26.7[1]	24.5	+2.2
	232				

[1] These numbers have the Mount Wilson results for the corresponding proper motions included in them.

This solution requires two corrections. The first is due to the difference in the average value of λ for the several groups. This correction has been applied on the very probable assumption that the change of ρ with λ is proportional to $1 + \cos^2 \lambda$. The second correction is due to the fact that while the values of μ are equal, or practically so, the values of π are somewhat unequal owing to the differences in magnitude. As a result the average linear motions and hence the radial velocities will be affected. This effect is systematic in character, though small, and can be applied only in the case of the K stars.

If we combine the results for the separate groups into means for each spectral type we obtain Table III. The corrected value for the K stars is denoted by $b2$. On the assumption, which seems probable, that the correction in the case of the other spectra is the same as for the K stars we find, therefore, *a change of radial velocity per unit of absolute magnitude of* 1.1 km. ± 0.4.

TABLE III. VALUES OF THE CONSTANT b IN KM.

TYPE	$b1$	$b2$	PROB. ERROR
F	+2.1	.....	±0.73
G	+1.0	.....	0.82
K	+1.6	+1.1	0.62
M	+2.6	.....	2.5
Mean	+1.6	.....	±0.40

The reality of this conclusion that the more luminous stars move more

slowly than the fainter stars must be accepted with considerable reserve. Certain features of this investigation are necessarily somewhat crude, and the data upon which it rests were not collected with a view to such a question. Moreover, of two direct observational points of evidence one appears to be opposed to this conclusion.

The stars denoted as *c* stars by Miss Maury were shown by Hertzsprung to be exceedingly luminous and very distant. According to an unpublished investigation by Kapteyn and Hertzsprung these stars, of average magnitude 4.5, must be 300 times more luminous and 4.5 times as far away as the average *A* stars of the fifth magnitude. We should, therefore, expect a low velocity in their case. In reality the average radial velocity of 28 of these stars is 12.8 km. For the average of all the *A* stars Campbell finds 11.1 km. In considering this result it should be borne in mind that the effect found here is for the second type stars and may not apply to the *A* stars; also that the *c* stars may actually constitute a separate spectral class with which the *A* stars are not directly comparable.

It is known from the researches of Hertzsprung that the variable stars of the *δ Cephei* type are stars of very high luminosity. He finds for their average absolute magnitude the value -2.2, using for this purpose their parallactic motions and apparent magnitudes. The average absolute magnitude of the 198 *F* stars given by Campbell is $+1.8$. The average radial velocity, freed from the sun's motion, of 11 of the *δ Cephei* variables, of average type *F*8, is 9.0 ± 1.2 km. The average radial velocity of Campbell's *F* stars is 14.75 km. This would give 1.5 km. as the value for our coefficient *b*.

SUMMARY OF RESULTS

1. The radial velocities furnish a very thorough test of the theory of the star streams. The results found for the *F*, *G*, *K*, and *M* stars are in close agreement with those we should expect from the theory as derived from proper motions.

2. The radial velocities of the stars of the smallest proper motions show the effects of the two star-streams with the same certainty as those of the other stars. The existence of the two star-streams is, therefore, proved at the greatest distances for which we have adequate data.

3. The *K* stars behave in general like the other stars, but there are a few exceptional cases. These do not appear to be due to the absence of the second stream.

4. For all of the spectral classes the average radial velocities show a regular increase with the proper motion.

5. Such a change of radial velocity is a necessary consequence of a velocity distribution (for the peculiar motions) different from that given by Maxwell's law.

6. A first approximation to the velocity distribution has been derived for the *K* stars. It explains the change of velocity with proper motion in a satisfactory manner.

7. Some positive indications have been found of a change of radial velocity with absolute magnitude, the brighter stars moving more slowly than the fainter stars.

[1] *Lick Obs. Bull.*, **7**, 113 (1913).
[2] See *Annals Harvard Coll. Obs.*, vol. **50**.
[3] Adams and Kohlschütter, "Some special criteria for the determination of absolute stellar magnitudes," *Contrib. Mt. Wilson Solar Obs.*, No. 89; *Astrophys. J.* **40** (1914).
[4] For example, *Astrophysical Journal*, **31**, 266 (1910).
[5] Kapteyn, *Monthly Notices*, **72**, 743 (1912).
[6] Eddington, *Observatory*, **34**, 355 (1911).
[7] *Proc. Amsterdam Acad. Sciences*, **14**, 524 (1911).
[8] Schwarzschild, *Astron. Nachr.*, No. 4557 (1912).
[9] See *Publ. Astron. Lab. Groningen*, No. 11, p. 12.

CRITIQUE OF THE HYPOTHESIS OF ANOMALOUS DISPERSION IN CERTAIN SOLAR PHENOMENA

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According to the theory of anomalous dispersion, the interpretations of solar observations in terms of absorption, motion, pressure, and level are generally misleading since the observed phenomena, such as prominences, flash spectra, flocculi, and displacements of the Fraunhofer lines are from this point of view mainly the effects of anomalous refraction in the solar atmosphere, so that in their study we are facing optical illusions. The establishment of such a point of view would revolutionize or render futile many of the present lines of solar and stellar observation, and would make practically impossible the solution of many problems which confront the investigator. It is therefore of the highest importance to determine the form and extent of the influence of anomalous refraction in the solar atmosphere, if it obtains to a detectible amount.

There is a degree of vagueness in the deductions from the theory, due to its extreme flexibility, that makes a quantitative examination of its claims difficult. During the present year, however, W. H. Julius, its

originator and chief supporter, announced a deduction that offers an opportunity of making such a quantitative test, namely: the mutual influence of Fraunhofer lines upon each other.[1] The observational data available for a crucial test are available,[2] and consist of the displacements of 506 Fraunhofer lines at the edges of the penumbra of eccentrically located sun-spots observed by me. In such observations the slit of the spectrograph is parallel to the radius of the solar image passing through a spot whose position for best observation is between the limb and the midpoint of the radius. The displacements are a maximum at the peripheral edges of the penumbra. At the edge directed toward the limb they are in general in the direction of longer wave-length, while at the edge directed toward the center of the disk they are in the direction of shorter wave-length. These displacements were interpreted by Evershed and myself as effects of motion in the line-of-sight, but are considered by Julius to be due to anomalous refraction.

In this phenomenon the particular form in which mutual influence would manifest itself is that a weak line on the violet side of and near to a stronger line is displaced less, but, if on the red side, more than the average amount. Julius, in making the announcement of the deduction to which reference is made, gave a discussion based upon a portion of the available data and obtained a result apparently confirming the deduction and considered by him a veritable proof of the theory. In fact, the deduction follows so directly from the theory that its confirmation would greatly strengthen the position of the theory as a working hypothesis, while a failure of confirmation would discredit it. The questions raised by Julius are elsewhere considered by me in detail.[3]

These displacements are well suited for a definitive test of the theory, as they are purely differential and free from possible observational bias, since they were made without any knowledge of what they are now used to prove. Moreover, a large number of other lines present upon the same plates are available for standards of reference. It is evident that much depends upon the derivation of these standards.

The 506 lines in my original paper do not form a homogeneous series of observations. From this it follows that standard displacements for different spectral regions cannot be determined with high precision by deducing them for each region by any smoothing-out process involving the results for all the segregated regions. There are six series:

(1) λ 3624 – λ 3724; (2) λ 3879 – λ 4410; (3) λ 4634 – λ 4829;
(4) λ 5123 – λ 5349; (5) λ 5598 – λ 6065; (6) λ 6393 – λ 6643.

The measurements represent with high precision the relative displace-

ments in any single region; but the data are not sufficient to obtain the relation between the absolute values for the separate regions to the same degree of precision, depending as they do upon varying observational conditions.

To obtain residuals capable of representing any systematic behavior of lines within the refractive influence of stronger lines, the standard displacements should be derived from the limited homogeneous series of observations in which they are to be used and, as far as the data permit, they should refer to the particular element under consideration. Such standard displacements have been obtained by determining the mean displacements for the lines of each element of a given intensity for each of the homogeneous series.

With these standards the displacements of 131 Fraunhofer lines within 0.5 A of stronger ones are compared, 67 on the violet side of stronger lines and 64 on the red side. The hypothesis of mutual influence requires that for the lines to the violet the mean residual be negative and for the lines to the red, positive. The result of the comparison applied to this large number of lines is that 67 lines to the violet give a mean residual − 0.0003 A, and 64 lines to the red give a mean residual − 0.0004 A.

Stated in another form, the sum of the favorable residuals is 0.212 A; of the unfavorable, 0.218 A. Tested, then, by standards deduced from the homogeneous series of observations to which they are to be applied, the lines under the influence of stronger ones show no systematic difference within the limits of measurement from lines not so situated.

Another criterion is whether the effects produced by the influencing lines depend upon their power to produce anomalous dispersion in the laboratory. A comparison of the residuals given by lines under the influence of those having strong anomalous dispersion with the residuals given by others under the influence of lines having very weak anomalous dispersion shows no greater effect in one case than the other.

It would seem that the effect of mutual influence must increase with nearness of the influenced line to the controlling line. The change should then be especially manifest for those in the near neighborhood of such powerful lines as H, K, and 4226.9 of calcium and the strong aluminum lines, all of which show large anomalous dispersion in the laboratory and are bordered by broad shadings or wings in the solar spectrum. From the point of view of the anomalous dispersion theory, these wings are considered dispersion bands. In making the original series of measures, the lines near these very strong lines were in general omitted from the observing list, as they present special difficulties. A

few of the best plates have now been remeasured with the view of obtaining data relative to such lines, and also a few plates of a new series of observations on spots belonging to the approaching spot-maximum. The displacements of 144 lines have been measured; of these 16 are within one angstrom of very strong lines showing great anomalous dispersion. For standards of reference the mean displacements for all lines of a given intensity are used. Within one angstrom of these strong lines there are eight lines to the violet which give a mean residual of + 0.0006 A based upon 51 measures, and eight lines to the red which show a mean residual of + 0.0003 A based upon 62 measures. The result is practically zero. There is, moreover, no systematic variation, as the influencing lines are approached from a distance of five angstroms to their near neighborhood.

The displacements so far considered are of lines in the reversing layer and indicate an outflow from the spot-vortex tangential to the solar surface. The important chromospheric lines, H and K of calcium, the hydrogen lines, the D lines of sodium and the b group of magnesium, give displacements of the opposite sign, interpreted as inflow of the high level vapors. Up to the present the theory of anomalous dispersion has not been able to suggest an explanation of these negative shifts.

All these displacements are obtained with the slit of the spectrograph parallel to the radius of the solar image passing through the spot. Displacements of the high level lines are also found when the slit is normal to the radius, which on the line-of-sight interpretation indicate vortex motion. These displacements have always been troublesome to the dispersion theory and are disposed of as occasional phenomena due to unequal refraction at the opposite edges of spots under exceptional conditions. The displacements are, however, a practically constant feature of all spots, persist for many weeks, and characterize regular spots during at least the greater part of their existence.

It is probably possible to imagine a density distribution capable of explaining these displacements, "Since almost any peculiarity in the appearance of spectral lines may be explained by anomalous dispersion, if only we are at liberty to assume the required distributions," as Julius remarks; but it seems questionable that such an artificial condition characterizes a regular spot and the mind accepts more readily the obvious explanation based upon motion in the line of sight.

There is one more consideration bearing upon an explanation of these displacements based upon the anomalous-dispersion theory, namely, the differences characteristic of the elements. In examining the standing of a theory, facts that cannot be explained by it are of particular

interest. The existence, then, of such differences becomes of great importance, since as yet it has not been possible to suggest an explanation of them in harmony with the dispersion theory. As standards of reference the displacements of the iron lines of like solar intensity measured upon the same plates have been used. The residuals (that is, displacement for the element minus the mean displacement of iron lines of the same intensity) are given in thousandths of angstroms in the accompanying table:

Element	Ba	Ce	Cd	La	Nb	Nd?	Pb	Yt	T
Number of lines	2	1	1	9	1	3	1	1	47
Atomic weight	137	142	112	138	94	144	207	173	48
Mean residual in 0.001A.	+5	+7	+5	+2	+10	0	+6	+6	−3

The residuals for the group of heavy elements are systematically positive and are undoubtedly real, while the residuals for titanium are systematically negative and likewise exceed the errors of measurement.

The displacements considered in this paper may all be interpreted in terms of motion and level as shown in contributions already published.[4] The purpose of the present discussion has been to show their bearing upon the theory of anomalous dispersion in its application to certain solar phenomena of which the theory has suggested partial explanations and to which it has appealed for evidence of its influence in the solar atmosphere. The only quantitative criterion, mutual influence, fails to confirm the theory, and in my opinion there are well established results of these observations for which the theory has no explanation.

[1] W. H. Julius: Radial motion in sun-spots. *Astrophys. J.*, **40**, 1 (1914).
[2] *Contrib. Mt. Wilson Solar Obs.*, No. 69.
[3] Ibid, No. 93.
[4] Ibid, Nos. 69 and 74; *Astrophys. J.*, **37**, 322 (1913); **38**, 341 (1913).

A REVISION OF THE ATOMIC WEIGHT OF CADMIUM

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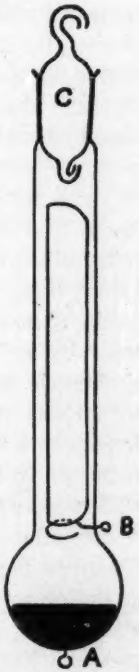
In several recent investigations by Hulett and his collaborators[1] concordant values have been obtained for the atomic weight of cadmium by the electrolytic deposition, in a mercury cathode, of the metal in hydrated and anhydrous cadmium sulphate and in anhydrous cadmium chloride and bromide, and by the comparison of simultaneous electrolytic deposits of cadmium and silver. The average value so obtained for the constant in question is 112.3. On the other hand, Baxter, Hines, and Frevert[2] had previously found a considerably higher value, 112.42, by the estimation of the halogen-content of cadmium chloride and bromide. So wide a discrepancy is disturbing and obviously needs attention. We therefore decided to repeat a portion of the work of Quinn and Hulett by analyzing anhydrous cadmium chloride through the electrolytic deposition of the cadmium in a mercury cathode.

Cadmium material was purified (1) by electrolytic deposition with a dissolving anode (yielding samples *A* and *B*), (2) by fractional precipitation with hydrogen sulphide (yielding samples *C* and *D*), and (3) by crystallization of cadmium bromide (yielding samples *E* and *F*); these operations being followed by crystallization of cadmium chloride (except in the case of sample *E* where the bromide was converted into chloride by fusion in a current of dry chlorine). Another sample (*G*) was obtained by combining the mother-liquors of the other specimens of cadmium chloride and, after crystallizing the salt, fusing the mixture in dry chlorine. In order to follow the purity of the different samples, the metal was deposited electrolytically from the mother-liquor of the last chloride crystallization in each case; and, after conversion into electrodes by fusion, it was used for the production of spark-spectra which were photographed in a Féry quartz spectograph. Careful examination of the spectrograms in the visible and ultraviolet regions showed that some of the samples were wholly free from impurities, and that, although some of them contained traces of copper and lead, the proportions of these impurities were too small to have any perceptible effect upon the outcome. Furthermore, the analyses of the chloride showed no apparent differences between the different samples. On the whole, crystallization of cadmium bromide seemed to be the most rapid and efficient method of purifying cadmium material.

For the electrolytic deposition of the metal the special form of cell shown in the figure was designed. In this cell both the cathode *A* and the anode *B* were fused into the glass and weighed with the cell, in order that any metal dissolved from the anode during electrolysis might immediately be deposited upon the cathode. This made it possible to electrolyze the chloride solution directly without previous removal of the chlorine. Another advantage of this form of cell lies in the possibility of introducing bodily the boat in which the salt was fused preparatory to weighing, so that no quantitative transference of solution was necessary. The mercury and the amalgams were prepared for weighing by washing with water and alcohol and drying in a high vacuum. Spurting of the amalgams out of the cell during the drying was prevented by the specially constructed stopper *C*, which was always weighed with the cell.



Blank determinations in which hydrochloric acid solutions were electrolyzed showed that the cell with a charge of mercury could be depended on to remain constant in weight. Furthermore, no evidence could be found of appreciable oxidation of the amalgams.

The experimental procedure of an analysis was as follows: The cadmium chloride, in a quartz boat contained in a quartz tube, was heated in a current of pure dry hydrochloric acid gas, gently at first so as to expel as much as possible of the water at a low temperature, but finally for some minutes at the fusing-point. After the tube had been allowed to cool, the hydrochloric acid was displaced by pure dry nitrogen, which was in turn displaced by dry air. Then the boat was transferred to the weighing bottle, in which it was originally weighed, by means of a 'bottling apparatus,'[3] without exposure to moisture, and weighed. Next the boat was inserted in the cell, in which it was supported upon the anode. The weighing bottle was rinsed, and the rinsings were used to dissolve the salt in the cell. For the purpose of retaining spray formed during the electrolysis, a column of bulbs, which had been ground into the mouth of the cell, was inserted. Electrolysis was then commenced, at first with a current of about 0.5 ampere. Later, when the greater part of the cadmium had been deposited, the current-strength was increased until finally the eight storage cells used as the source of electricity were short-circuited through the cell. The whole operation ex-

tended over about eight hours. At the end chlorine had nearly ceased to be evolved at the anode, although some residual conductivity remained, owing to the formation of a small amount of perchloric acid. Toward the end of the electrolysis the column of bulbs and the walls of the cell were several times rinsed into the cell. At the completion of the electrolysis the electrolyte was displaced with water which had been freshly saturated with hydrogen; and the water in turn was displaced with pure alcohol, the maximum voltage being maintained between the electrodes during the operations, and care being taken not to break the electrolytic circuit. The cell was then placed in a desiccator containing fused potassium hydroxide, the pressure in the desiccator was reduced to a few millimeters of mercury with a mechanical pump, and the cell was allowed to stand until the alcohol had evaporated. The cell was next transferred to a second desiccator which contained the counterpoise of the cell, and the two were allowed to stand in a low vacuum for some hours before being weighed. Further standing in an exhausted desiccator produced no change in weight, showing both that the cell and amalgam were dry and that the amalgam had little tendency to oxidize. The rinsings, both aqueous and alcoholic, were evaporated in a quartz dish with a very small amount of redistilled sulphuric acid; and, after transference to a weighed platinum crucible, the residue was ignited at dull redness. In this way a residue, in most cases weighing less than one milligram, was obtained which was shown to be cadmium sulphate. A correction to the weight of cadmium was applied upon this basis. Complete precipitation of the cadmium was never secured, although, according to the experience of Quinn and Hulett, no cadmium is left undeposited, even when a relatively large amount of sulphuric acid is present. The small amount of free chlorine contained in the residual electrolyte in our experiments may have been the cause of the incomplete deposition of the metal.

The weighing-bottle containing the boat and the electrolytic cell were both weighed by substitution for a counterpoise very similar in construction and of the same total volume.

The experimental data and the results calculated from them (assuming the atomic weight of chlorine to be 35.457) are shown in the accompanying table. The weights here given have been reduced to the vacuum standard by applying the following corrections for each gram of substance: cadmium chloride, + 0.000152 g, cadmium dissolved in mercury, − 0.000016 g.

It will be seen that this research has yielded a value (112.417) for the atomic weight of cadmium in very close agreement with the earlier one

(112.42) obtained by Baxter, Hines, and Frevert. In other words the determinations of the cadmium and chlorine in cadmium chloride sum up to almost exactly one huudred per cent. The disagreement of the outcome of this research with that of Quinn and Hulett's work also is marked.

ANALYSIS NUMBER	SAMPLE	GRAMS OF $CdCl_2$	GRAMS OF Cd	RATIO $Cd:Cl_2$	ATOMIC WEIGHT
1	A	6.08570	3.73181	1.58538	112.426
2	A	4.20489	2.57863	1.58562	112.443
3	A	5.36203	3.28817	1.58553	112.436
4	B	7.50512	4.60221	1.58538	112.426
5	B	6.71591	4.11839	1.58551	112.435
6	B	5.91556	3.62763	1.58555	112.438
7	C	5.49323	3.36805	1.58483	112.387
8	B	3.14416	1.92791	1.58513	112.408
9	C	7.58705	4.65173	1.58474	112.381
10	B	2.26738	1.39006	1.58444	112.359
11	B	5.93501	3.63962	1.58562	112.443
12	F	6.04122	3.70490	1.58578	112.454
13	D	4.07400	2.49821	1.58537	112.425
14	E	9.00004	5.51879	1.58529	112.419
15	G	6.56891	4.02808	1.58534	112.423
16	G	7.12956	4.37174	1.58522	112.414
17	E	8.57291	5.25679	1.58522	112.415
18	G	7.76294	4.76011	1.58521	112.413
Average				1.58529	112.419
Average of analyses 14 to 18				1.58526	112.417

It is a great pleasure to express our gratitude to the Carnegie Institution of Washington and to the Elizabeth Thompson Science Fund for very generous assistance in providing indispensable apparatus.

[1] *J. Phys. Chem.*, **15**, 1579 (1911); *Trans. Amer. Electrochem. Soc.*, **22**, 385 (1913); *J. Phys. Chem.*, **17**, 780 (1913).

[2] *J. Amer. Chem. Soc.*, **27**, 222 (1905); **28**, 770 (1906).

[3] Richards and Parker, *Proc. Amer. Acad.*, **32**, 59 (1896).

PHORADENDRON

By William Trelease

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Presented to the Academy, October 30, 1914

Several years ago my attention was seriously called to the need of a revision of our leafy mistletoes through inability to understand the basis of characterization that could admit to one species such different appearing plants as those from the southeastern, southwestern, and arid United States—not to mention California and Yucatan—to which the name *Phoradendron flavescens* is currently applied Among their manifold differences, a diligent search was made for characters; the types of related species and varieties that have been held to be differentiable from *flavescens* were examined with care, and every form occurring in the United States was traced to the known limits of its range, sometimes south of our national border. As political boundaries do not often form satisfactory limits to such a study as I had begun, I was quickly lured into an examination of the Mexican species which approach our border, and of others which reach into the field of these, so that no arbitrary geographical limit, even, could be fixed short of the Isthmus.

In the course of this study it became apparent that the great conservatism of Engelmann, who seems never to have given this genus the care that marked his study of the related genus *Arceuthobium* or *Razoumofskya*, had not only caused him to withdraw segregates of *P. flavescens* that he admitted at one time, but had reacted on his early colleague in the study of our southwestern plants, Torrey, to the extent of causing a number of forms which had been designated in the Torrey herbarium as new species to lie there, as they still do, without publication.

At the New York meeting of the National Academy of Sciences, in November, 1911, and at a meeting of the Academy of Science of St. Louis on December 18, 1911, the preliminary results of this study of the northern species were outlined, and this was followed at the Washington meeting of 1912 by presentation to the National Academy of the manuscript of a revision of all of the forms of *Phoradendron* recognized as occurring in continental North America. As I was then on the eve of departing for a year in the great herbaria of Europe, this revision was withheld from immediate publication in the hope that several obscure Mexican species could be cleared up certainly, through authentic specimens, and in the hope that they might be illustrated from the types. Though the admission of Torrey's long neglected manuscript names

had quite prepared me for an apparently inordinate increase in the number of differentiable species in the genus, I was not a little surprised to find, when casting my results then into classified form, that on an average nearly two new named forms appeared for each one already admitted to our flora.

Notwithstanding an intention to limit my investigation to the species of continental North America, the temptation to learn the characters of the South American species proved irresistible when, at Brussels, I examined the specimens in the personal herbarium of von Martius, whose collections have done so much to make known the flora of Brazil; and it was not long before the genus as a whole engaged my attention, though West Indian material was given less care than the other until at Dahlem I reached the collection of Professor Urban, who in 1897 had published a revision of all of the West Indian Loranthaceae. To my keen satisfaction, I then found that for the Antillean region very few forms were to be differentiated from those admitted by Urban, confirming my judgment that the large increase in our own flora rested upon previous neglect of application to them of characters which appear to be really differential, rather than to excessive optimism concerning their separability. The thorough study of tropical forms by Eichler in his revision of Loranthaceae for the Flora Brasiliensis, in 1868, supplemented by a reëlaboration of available material when Urban monographed the West Indian forms, has also prevented an increase in the number of South American species at all comparable with that within our own flora, though the number of names added is relatively greater than for the Antilles.

The essential characters of *Phoradendon* in its group of Loranthaceous genera are chiefly its axillary spikes of sessile unisexual and monochlamydeous small flowers often sunken in hollows of the frequently swollen internodes of the rachis, normally trimerous, with 2-celled longitudinally dehiscent anthers. A very few species, like *P. cymosum*, present the phenomenon of a terminal spike corresponding to the 1- or few-flowered cyme of the old world *Viscum*, but in addition to axillary spikes. Except in the species taken by Hooker for *falcatum*, the receptacular cups, which range from so shallow as hardly to surround the base of the flower to a depth covering a noticeable part of the mature fruit, are essentially even on their margin; but in this species the cup is parted so as to present sometimes the appearance of a deeply divided calyx. The flowers, with a small vestigial nectar-gland and apparently adapted to pollination by such short tongued insects as flies and small bees, are usually yellowish green when expanded, but in *P. Brittonianum* and some of

its relatives the sepals are blood-red even before anthesis. Some species are known to be apogamous and apogamy is to be expected in many others, a circumstance very probably connected, as in *Taraxacum* and *Hieracium*, with polymorphism or close affinity in species as now understood.

In all of our own species the plants are strictly dioecious and, as a rule, staminate spikes are longer than pistillate and bear more flowers. This is known to be true also of a number of tropical species, such as *P. Wattii*: in others, prevailingly if not exclusively staminate and pistillate spikes showing something of the same dimorphism occur monoeciously on the same plant. Though usually not too closely applicable as between related species, the number and arrangement of the flowers on a given spike present equally characteristic differences, but with the qualification that flowers of the uppermost joints may be fewer in number and simpler in grouping than below, while one or two of the lower joints may be partly or entirely without flowers, the lowermost almost universally being reduced to a sterile peduncle. The greater number of tropical species differ from those of the north in being androgynous through the occurrence of a number of staminate flowers on spike-joints that are otherwise pistillate, or, less commonly and sometimes differentiated by the term 'gynandrous,' through the occurrence of a few pistillate flowers on otherwise staminate joints,—as many of Eichler's accurately drawn plates show very beautifully. Except in a broad way, these differences do not appear to be practically applicable in contrasting species though representing in part morphological differences of fundamental taxonomic value. The prevailing grouping of the flowers is in 2, 4, or 6 series on each joint of the spike; i.e., in 1, 2 or 3 ranks over each of the two scales by which it is subtended. Examples of the first and last are given by *P. laxiflorum* (2), and *P. flavescens* (6), and where the joints are unisexual these numbers commonly prevail, though four series may be found by reduction and as many as ten by increase when the number is typically six. When the joints are androgynous, the staminate flowers often occur at top between the normal two ranks over each scale, and this condition is usually accentuated on luxuriant spikes and sometimes on all by the downward intrusion of a partial or complete third series over each scale. For the separation of the groups into which tropical species fall, I have found it most convenient to use the prevalence of 2 or 6 series of flowers on the joint as a differential, providing as an intermediate the prevalence of the interjected two series under the designation 4 + 2. A glance at *P. domingense* (2), *P. trinervium* (4 or 4 + 2), *P. hexastichum* (6) and *P. Lindavianum* (6–10) will make these distinctions evident,—

more than 6 ranks being very exceptional except in some tropical species with leaves venulose above and dull beneath, and in some of our northern forms.

One of the characters most available and significant in the classification of the species of *Phoradendron* is a fundamental difference in their leaves. By far the larger number of species have unmistakable leaves, but our western group to which *P. californicum* and *P. juniperinum* belong have their foliage reduced to short thin scales which resemble the leaves of the related genus *Arceuthobium* or *Razoumofskya* so closely that species of either genus are commonly to be found in herbaria as representative of the other. Unlike typical foliage leaves, these scales do not disarticulate, though a constriction at the base of the scales in two forms affords partial ground for their specific recognition: one species of the Mexican mountains, *P. minutifolium*, has almost equally small though fleshy disarticulating leaves, and two of the South American species, *P. tunaeforme* and *P. fragile*, are characterized by bearing small scale-like leaves only. Such species are very likely to be mistaken for some representatives of the related genus *Dendrophthora*, which differs technically in its 1-celled anthers.

If any species of the United States, for example *P. Eatoni* of the everglades of Florida, is compared with any West Indian or South American species, for example *P. rubrum* of the Bahamas, the latter will be found to possess constantly in addition to its foliage one or more pairs of scale-leaves at least on the lowermost foliage internode of every branch. Comparable with the scales of the flowering spikes and with the stem-scales of *P. juniperinum*, etc., these cataphyls afford by their presence or absence what proves to be one of the most important of characters for the primary division of the genus *Phoradendron*. Usually cataphyls do not subtend flowers or spikes, apparently serving no function further than the protection they may afford the shoot in its earliest development; but in *P. crassifolium* and *P. craspedophyllum* spikes are regularly and characteristically found in the axils of some of the cataphyls, and less characteristically in a few other cases.

Never found on any species of the United States, absent from three-fourths of those of Mexico and Central America, but invariably present on all of the South American and West Indian species, these scales are usually confined in the latter to the basal joint of each branch, though in cases of true or cymose forking they are found on all joints—since only basal joints are then present. In a very small percentage, only, of the tropical species with percurrent or monopodial branching, e.g., *P. flavens* and *P. crassifolium* and their allies, cataphyls are found on all

foliage internodes; and in a single known species, *P. paradoxum*, the stem is made up of rather terete joints with cataphyls and ancipital joints without them, in regular alternating succession.

In the geographic distribution of its species, *Phoradendron* is rather unusually instructive. The genus is strictly American and extends from Washington, Colorado, the mouth of the Ohio River, and southern New Jersey to the Southern Argentine region on the continent, and through the entire West Indian chain; one species occurs in the Pacific island Guadalupe, and two are found in the Galapagos group of Pacific islands,—both oceanic but with American floras. None of its many species of fairly homogenous character possesses a very wide geographic range. Marked examples of widespread occurrence are afforded only by a few such polymorphous species as what is usually called *P. latifolium*, or an assemblage of intricately related if differentiable species like that usually known as *P. rubrum* or *P. quadrangulare*, both of which range from Brazil to Central Mexico and well through the West Indies. Few species, indeed, equal in range our native *P. flavescens*, which occurs from southern New Jersey to the lower Wabash, Oklahoma and eastern Texas, reaching southeast to the Gulf and ocean.

Admirably endowed with means of free dissemination through its edible berries with extremely viscid pulp, which leads to their dispersal by birds, these mistletoes seem limited nevertheless to a surprising extent by ordinary barriers to plant migration. Like the similar European *Viscum album*, with its scarce-definable races capable of effective germination only on the host-species from which the seed came, our eastern *P. flavescens* though attacking a large variety of host plants is usually found confined to a single host in a given region, and such experiments as have been made on it show that it can be transferred from one host to another with difficulty if at all. How far this may be concerned in the polymorphism of this species and how far its like may serve to limit the dispersal of most species is at present a matter of conjecture only.

Viewed on broad geographic lines, the species of *Phoradendron* usually occupy areas that present severally an assemblage of fairly uniform meteorologic features with limiting environment,—in this respect agreeing with most other plants and with animals. In the main, the regions affected by individual species of *Phoradendron* are the following:

North America: (1) Atlantic, (2) Rocky Mountain, (3) Pacific, (4) Great Basin or Sonoran,—in the United States; (5) Table-land, (6) Eastern or Western Sierra Madre, (7) Western riparian, (8) Yucatecan, and (9) Cordilleran,—in Mexico and Central America.

South America: (10) Andean, (11) Peruvian and Bolivian, (12) Argentine, (13) Brazilian upland, apparently with less differentiation than would be thought between the arid and humid portions, (14) Amazonian, (15) Cayenne, and (16) Venezuelan.

West Indies: (17) Caribbean, (18) Antillean, to the north of the Anegada passage, and (19) Bahamian.

Few species range throughout any one of these regions, and it is very rare for a species to reach from one into the other.

Briefly summarized, the purely taxonomic part of my study of the genus leads to the conclusion that *Phoradendron* may be best divided into two primary groups, respectively constantly without and constantly with cataphyls on their foliage shoots: for the first I am using the name BOREALES, since its species alone are represented in the north; and for the other, AEQUATORIALES, since its species only are found in the equatorial region. Species destitute of expanded foliage are found in each group in small numbers. Those of the first group are pubescent for the most part, while only two of the second group are more than papillately roughened. The Boreales appear to be strictly dioecious; the Aequatoriales for the most part, though not exclusively, are monoecious, usually with some or all of their spikes androgynous.

So far as shown by the material now contained in the great herbaria at Washington, New York, St. Louis, Chicago, Brussels, Copenhagen, Kew, Munich (where von Martius' official collection is), Geneva, Buda-Pest, Prag, and Dahlem, and in many smaller collections, I find a total of 262 differentiable forms, most of which I regard as species, and of which 60, or 2 3 percent, are of the Boreales and 202, or 77 percent, Aequatoriales. Of the Boreales, 41, or two-thirds, and of the Aequatoriales, 85, or two-fifths, are characterized as new.

The distribution of the main groups (species which occur in more than one region being included in each) is:

	U. S.	*Mexico*	*Centr. Amer.*	*W. Indies*	*S. Amer.*
BOREALES	26	45	1	0	0
AEQUATORIALES	0	25	23	34	124
	26	70	24	34	124

THE MORPHOLOGY AND RELATIONSHIPS OF PODOMITRIUM MALACCENSE. (STEPH.)

By Douglas H. Campbell

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There are but two species of the liverwort, Podomitrium, *P. phyllanthus*, a common species of the Australian region, and *P. malaccense*, which according to Stephani is known only from Singapore and New Caledonia. The writer collected the latter species in Sarawak, Borneo, in February, 1913, and also at one station in Luzon, in May of the same year. The plant is common in Sarawak, and sufficient material was secured to make a pretty complete study of its structure, which has not been done before.

Podomitrium is often included in the genus Hymenophytum, but it is certainly generically distinct.

The sterile plants of *P. malaccense* resemble exactly certain species of Blyttia, and the plant can only certainly be recognized when it bears the reproductive organs. Fertile plants, however, are immediately distinguishable by the position of the reproductive organs, which instead of being borne on the dorsal surface of the ordinary shoots, are formed upon the special branches which grow from the side of the midrib upon the ventral side of the main shoot, very much as in Metzgeria.

As in Blyttia, the thallus has a very conspicuous, and sharply delimited midrib, and wings which consist of a single layer of cells. In *P. malaccense*, the thallus is usually unsymmetrical, one of the wings being almost obliterated. Rather short rhizoids occur upon the ventral side of the midrib, but these are not uniformly distributed. They occur in dense patches separated by considerable intervals which are quite free from rhizoids.

In the form of the apical cell, and in the character of the thick-walled cells which occupy the axis of the midrib, Podomitrium closely resembles Blyttia.

The fully developed plant may reach a length of about 4 cm. with a maximum width of about 4.5 mm. No cases of dichotomous branching were seen, all of the branches appearing to be of adventitious origin, springing like the sexual shoots from the sides of the midrib.

The antheridia arise in somewhat irregular rows, one on each side of the midrib of the special shoot upon which they are borne. They arise in acropetal succession, and each is covered by a scale with laciniate margins.

Both in structure and position the antheridia are much like those of some species of Blyttia, and perhaps still more like those of *Calycularia radiculosa*.

The archegonial branches are very short, and bear but one receptacle which is relatively very large. The receptical is very much like that of Blyttia, having a similar large involucre, within which is formed a second envelope, the perianth. The latter can be seen in the young receptacle as a shallow ring enclosing the group of archegonia. If one of these is fertilized, the perianth begins to grow very rapidly, and soon becomes a conspicuous tubular sheath enclosing the developing sporophyte.

The young embryo shows a conspicuous suspensor, and very closely resembles corresponding stages of the embryo of *Blyttia radiculosa*. As it develops, however, it is more like the embryo of Mörkia or Calycularia. This is shown in the development of a distinct foot, which is not so evident in Blyttia, and the capsule is less elongated than in Blyttia, resembling more nearly that of *Calycularia radiculosa*, with which it also agrees in the dehiscence of the capsule. In *P. malaccense*, the capsule opens by two completely separate valves, while in Blyttia (and it is said also in *P. phyllanthus*), there are four valves which remain united at the apex.

In the structure of the wall, and the markings of the spore membrane, *P. malaccense* much resembles a typical Blyttia.

It is evident that Podomitrium is much more like Blyttia, than like Metzgeria, and should be placed in the Blyttiaceæ, rather than Aneuraecæ.

THE FEEBLY INHIBITED. I. VIOLENT TEMPER AND ITS INHERITANCE

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Presented to the Academy, November 12, 1914

Recent studies have shown that the impulse to general mental development and the nervous 'strength' which successfully resists the stress of untoward conditions and emotional shocks have a clear hereditary basis. The hereditary factor behaves, indeed, in typical fashion. But it has often been pointed out by sociologists, who for the most part hold differences in 'conditions of life' responsible for the differences between men, that there is no evidence that the elements of moral or social behavior have a hereditary basis, and while there have not been

wanting those who have insisted that 'criminality is hereditary' yet no one has successfully determined the method of such inheritance. It appears, indeed, improbable that so complex a thing as criminality should prove to have a single hereditary factor. An attempt has been made to study the hereditary behavior of some of the elements of moral action—to analyze the family history of persons who have marked emotional traits.

Opportunities for such study have been afforded by 165 family histories of wayward girls in state institutions gained by trained field workers who visited the homes of the patients and got as full an account as possible of the behavior of all of the close relatives. The data were gathered without prejudice; indeed, it was impossible for the 'field worker' to know what laws of inheritance the histories might yield. Also, many other sources of information at the Eugenics Record Office have been drawn upon for additional data as to the inheritance of special traits.

Violent and more or less periodic outbursts of temper occur in families which are characterized by prevalence of epileptic attacks, also in those exhibiting cases of mania, also in others in which 'hysterical' attacks are common. The special form of the attacks differs somewhat in these classes of families, but the method of inheritance of the tendency is the same in all, and it seems probable that in each class the simple symptoms of the emotional outburst are modified by the differences in these three classes of the nervous condition.

The method of inheritance is indicated at once by the fact that, in the 66 family histories studied, the tendency to outbursts does not, typically, skip a generation. In one history it is traced through 5 generations; in a large proportion of the histories it is traced through 3 consecutive generations. The few cases in which neither parent of an affected individual is reported to have the tendency to outbursts are explained by obvious insufficiency in the record.

The fact that the tendency to outbursts of temper does not skip a generation indicates that it is a positive or dominant trait. That segregation of this tendency occurs is shown by the ratio of affected offspring in any fraternity to the total number of offspring whose emotional history is fully described. From the mating of an uncontrolled and a normal person expectation is that 50 percent of the children will be uncontrolled. A summation of all such children gives a total of 106 affected among 219 sufficiently described, or close to the 50 percent expected on the hypothesis that the tendency to outbursts of temper is a simple, positive trait.

The detailed investigation will appear in the *Journal of Nervous and Mental Diseases* and in the *Bulletin of the Eugenics Record Office.*

THE ENGLISH RABBIT AND THE QUESTION OF MENDELIAN UNIT-CHARACTER CONSTANCY

By W. E. Castle and Philip B. Hadley

LABORATORY OF GENETICS OF THE BUSSEY INSTITUTION, HARVARD UNIVERSITY, AND OF THE AGRICULTURAL EXPERIMENT STATION OF THE RHODE ISLAND STATE COLLEGE

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Whatever the theoretical importance of Mendel's law, its practical utility depends largely upon the purity of the gametes. If Mendelian unit-characters can through hybridization be recombined in desirable ways *without essential modification* during the process, Mendel's law is evidently a distinct acquisition to the practical breeder. Nevertheless if crossing is likely to produce considerable changes in the characters which it is desired to combine in a new race, it is evident that Mendelian crosses must be used judiciously and with caution by the practical breeder.

Considerations such as these have led W. E. Castle for several years to concentrate his studies of genetic problems upon the question of gametic purity. As a crucial experiment he conceived the plan of deriving an entire race of animals, not from a single *pair* of ancestors, but from a single gamete, so far as concerns a particular unit-character. It was thought that in a race so derived, if the principle of gametic purity holds, there should be no variation whatever in the particular unit-character concerned.

Color patterns of mammals seemed especially well adapted for such studies, since they are early differentiated and clearly Mendelize in crosses. The so-called 'English' piebald rabbit presents an especially fine example of such a color pattern. The figures give a good idea of this striking pattern in which white and colored areas are interspersed much as in the 'coach-dog.' It would be a distinct gain to breeders if they could reduce the variation in details of the English pattern so that 'prizewinners' could be bred without the production of so many 'wasters,' which depart in essential points from the standard pattern adopted for the breed. This was an additional reason for undertaking work with the English rabbit.

The first standard-bred English rabbits which Castle had under observation, when mated *inter se*, produced young of three sorts. About half the young were fairly good 'standard' English extensively marked with colored spots (see fig. 3). About one-fourth were much whiter than the standard demands, their spots being fewer and smaller (see fig. 1). And the remaining fourth were without spots, that is were

self colored. This last class was found to be recessive and not to produce English offspring, if mated *inter se*.

The *whiter-than-standard* English proved to be homozygous for the pattern, the 'standard' English being heterozygous and breeding like their parents.

From these observations it was clear (1) that the English pattern is a Mendelian dominant and (2) that the breeding of English rabbits resembles that of blue Andalusian fowls. For the standard bred animal is a heterozygote in the production of which there is bound to be a constant production of 'wasters' unless either the standard is changed or the homozygote can be changed to conform with the standard, producing an animal with more color. In the latter case homozygotes could be bred with each other and wasters eliminated. The question whether the pattern can be changed becomes therefore one of practical as well as theoretical interest.

In making crosses of English with other breeds of rabbits, there was found to be considerable variation among the heterozygous English produced, some being much whiter than others; i.e., having less extensive colored spots. Plus (dark) and minus (light) selections were made to see to what extent the pattern was capable of modification. These selection experiments are still in progress but will be reported upon at another time.

The single-gamete experiment, with which this report will deal, was placed in the hands of P. B. Hadley, who has carried it out at the Rhode Island Agricultural Experiment Station.

As foundation stock for the experiment a single *heterozygous* English rabbit of standard character (grade 2, fig. 5) was selected. To mate with him, it was desired to obtain a distinct breed of rabbits, free from the English pattern, and as pure (uniform) in all respects as possible. For this purpose the 'Belgian hare' was chosen. A buck and two does obtained from G. W. Felton, Cliftondale, Mass., were found to breed very true. From them was bred a stock of does very uniform in character, twelve of which, together with one of the parents (2A) were mated with the selected English buck which we may henceforth call by his record number ♂21A. The young thus produced will be called 'Series I' offspring. About half of them were self (non-English), the remainder (187 in number) were English.* The latter, although all

* The total number of young obtained from ♂ 21A, when mated with Belgian hare does has been to the time of writing 436. The English young now number 210, the non-English (self) number 226. For Series II matings presently to be described the corresponding numbers of young are: English 219, non-English 196, total 415. For Series I and II combined the numbers are : English 429, non-English 422, total 851. This is unmistakably a 1 : 1 Mendelian ratio.

undoubtedly heterozygous, varied in whiteness from grade 1 to grade 4, figures 1 to 4, the *modal* or commonest condition being about the same as that of the father (grade 2). The distribution of the young in relation to our grades is shown in the Table. Statistical treatment of the table gives the *average grade* of the young as 2.43, that is some-

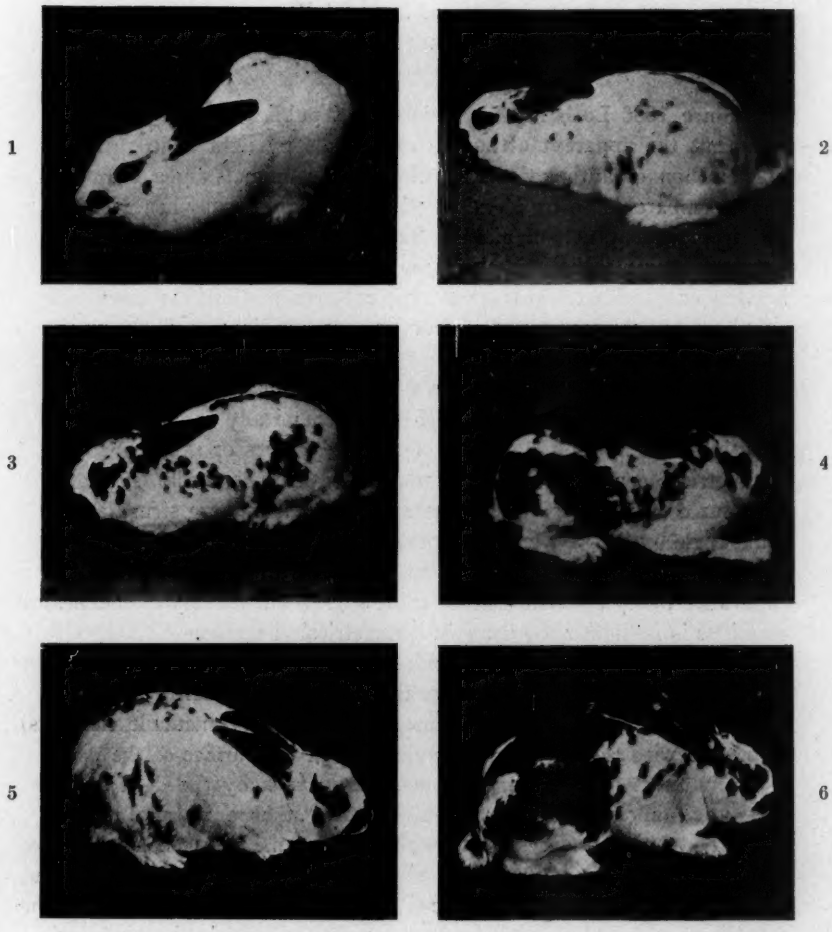
TABLE SHOWING THE DISTRIBUTION OF GRADES OF OFFSPRING IN THE FIRST AND SECOND SERIES OF MATINGS FOR EACH INDIVIDUAL MOTHER

MOTHER	SERIES	GRADES OF YOUNG													TOTALS		AVERAGE
		1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	Ser. I	Ser. II	
16A	I					1	2	2							5		2.30
	II					**3**	**1**	**1**	**2**	**1**	**7**	**3**	**3**			**21**	**3.03**
16B	I		1		1	3	1	1	1	2			1		11		2.39
	II			**1**	**1**	**2**	**1**		**1**	**2**	**1**	**3**				**12**	**2.67**
16D	I					2	5			1	2	3	1		14		2.79
	II								**3**	**1**	**6**		**2**			**12**	**3.19**
16E	I			2		5		2	1	1			1		12		2.29
	II										**1**	**3**	**1**			**5**	**3.50**
16F	I	1				2	1	5	3	2		3			17		2.62
	II				**1**	**2**	**1**	**3**	**3**			**1**				**11**	**2.48**
16G	I		1	2		2	3	1	2	3	1				15		2.35
	II						**1**	**2**	**2**	**3**	**1**	**2**	**1**	**1**		**13**	**3.06**
16H	I			1		3	3	1	2	2	2	1	2	1	18		2.76
	II					**3**		**2**	**1**	**1**	**8**	**2**	**2**	**1**		**20**	**3.06**
17E	I		1	1	3	4	3	3	1	1	2	2			21		2.36
	II					**1**	**2**	**5**	**5**	**3**	**4**	**4**	**2**	**1**		**27**	**2.97**
17G	I			1	9	6	3	3	2	1	2	1	1		29		2.27
	II		**1**	**1**		**1**	**1**	**1**	**1**	**1**	**1**	**1**				**9**	**2.44**
18D	I		1			3	3	3		1	2	2			15		2.53
	II					**2**	**1**	**2**	**3**	**1**	**2**	**5**				**16**	**2.91**
18F	I				3	1						1			5		2.15
	II				**1**	**1**	**2**		**4**	**2**	**1**	**1**	**2**	**2**		**16**	**2.97**
18H	I			1	1		3	1	1	3					10		2.43
	II				**1**		**3**	**3**	**4**	**2**	**3**		**1**	**2**		**19**	**2.87**
2A	I		1	2	1	1	4	2	3	1					15		2.22
	II				**1**	**2**				**1**	**2**	**2**				**8**	**2.78**
Totals	I	1	5	10	18	33	31	24	16	18	11	13	6	1	187		2.43
	II		**1**	**2**	**5**	**17**	**13**	**19**	**29**	**18**	**37**	**27**	**14**	**7**		**189**	**2.92**

what *darker* than the father. Inspection of the table shows that more than half of the young are darker than the father, which supports in a general way the statistical average grade. If we consider separately the average grade of the young produced by each mother, we find that it ranges from 2.15 in the case of ♀18F, which had 5 English young, to 2.79 in the case of ♀16D which had 14 English young. The average number of young to a mother is 14.4.

After this series of matings had been completed, a second series was begun in which the same 13 females were mated with one of the darkest bucks produced in the Series I matings (a son of ♀16 E). The selected buck was ♂40A (fig. 6) grade 3.75, considerably darker than his father (fig. 5). This series of matings produced 189 English young, together with a like number of self (non-English) young. The grade distribution of the English young is shown in Table 1, Series II. All of the 13, mothers except one (♀16F) produced darker offspring in the Series II than in the Series I matings. The lowest average grade was shown by the young of ♀17G, viz. 2.44. For Series I matings, the lowest average was 2.15. The highest average grade in the Series II matings was given by the young of ♀16E, viz. 3.50. For Series I matings, the highest average was 2.78. Consequently both maximum and minimum averages were higher in the Series II than in the Series I matings. The grand average of all the 189 Series II offspring was 2.92, as compared with 2.43, the average grade of the Series I young. The modal grade of the Series II young is 3.25. The modal grade for Series I was 2.00. Since the mothers were identical in both series, the difference in the young can be attributed only to the difference in the fathers. The male used in the Series II matings differed genetically as well as somatically from his father who sired the Series I young. Not only was he darker, but he also produced darker English young. Yet the father contained only a single dose (one gamete) of English pattern and the son derived his English pattern exclusively from this same source. Hence the English unit-character had changed quantitatively in transmission from father to son. This seems to us conclusive evidence against the idea of unit-character constancy, or "gametic purity." If unit-characters are not constant, selection reacquires much of the importance which it was regarded as possessing in Darwin's scheme of evolution, an importance which many have recently denied to it.

The question whether an imaginary 'unit-factor' for English pattern has or has not changed in correlation with the visibly changed English unit-character is not here discussed. We recognize that it has an academic interest, which however scarcely affects the practical question whether the visible Mendelizing characters of animals are subject to change through crossing or through selection or both.



FIGS. 1-4, PHOTOGRAPHS OF FOUR ENGLISH RABBITS FORMING A GRADED SERIES IN AMOUNT OF COLORED FUR.

These were adopted as our standard Grades 1-4, in terms of which all the rabbits described in these experiments have been classified.

FIG. 5, RABBIT ♂21A, FATHER OF THE SERIES I YOUNG.

FIG. 6, RABBIT ♂40A, FATHER OF THE SERIES II YOUNG.

ON THE IDENTITY OF HELIOTROPISM IN ANIMALS AND PLANTS

By Jacques Loeb and Hardolph Wasteneys

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1. Since 1888 Loeb has in a number of publications expressed the idea that the apparent attraction of animals by light is in reality a phenomenon of automatic orientation of the animals by a source of light, comparable to or identical with the well-known phenomena of heliotropic orientation of plants. Thus he proved that sessile animal organisms react to light in the same way as sessile plants, namely, by bending or growing towards (or away from) a source of light; while motile animals move towards (or away from) a source of light as do the motile swarm spores of certain algae.

He based his theory of these reactions in animals on the following three assumptions. First, the light acts chemically upon the photosensitive elements of the surface of the body (eyes or skin). Second, symmetrical elements of the surface are identical chemically, so that if one source of light is given and symmetrical elements are struck at the same angle by the rays emanating from one source of light, the velocity of the photochemical reactions in the symmetrical elements is identical; if, however, they are struck at different angles, the velocity of chemical reactions is no longer the same in symmetrical elements.

Third, the velocity of photochemical reactions in the eyes or the skin influences (through the nerves or, as the case may be, other protoplasmic conductors) the tension of the muscles (or other contractile elements) connected with the photosensitive elements at the surface of the body. If symmetrical photosensitive elements are struck by the light at the same angle (and only one source of light exists), the symmetrical muscles of the body are influenced by the light in the same way and no change in the direction of the motion of the animal will occur. If, however, symmetrical photosensitive elements are struck at different angles the velocity of chemical reactions will not remain the same in the symmetrical elements and hence the tension of symmetrical muscles connected with these elements will not be the same; as a consequence, when the animal moves it must show a tendency to deviate from the straight line until finally its axis or plane of symmetry goes through the source of light again. When this happens the symmetrical elements of the photosensitive surface are again struck at the same angle by the light and there is now no more reason for the animal to deviate from this direction.

If this theory of heliotropism were correct, it was necessary to show that the law of photochemical action should hold for the heliotropic reactions of plants as well as of animals. This law (which was first established by Hankel for a narrow range of light intensity and extended by Bunsen and Roscoe over a wider range) says, that within certain limits the photochemical effect of light is equal to the product of intensity into the duration of illumination. Five years ago it was shown independently by Blaauw[1] and by Fröschl that the heliotropic reactions of plants obey the law of Bunsen and Roscoe, i.e., that the time required to bring about the heliotropic curvature of plants changes inversely with the intensity of illumination. A year ago Ewald and Loeb[2] showed that the same law holds also for the heliotropic curvature of an animal, namely, that of the polyps of *Eudendrium*.

2. Although the question of wave length does not enter into Loeb's theory of heliotropic orientation, it seemed of interest to compare the relative efficiency of the various parts of the spectrum upon the production of heliotropic curvatures in *Eudendrium* with that found for the heliotropic curvatures in plants. The most exact measurements on plants are those by Blaauw on the seedlings of *Avena*. Blaauw used a carbon arc spectrum, and found that the most efficient part of the spectrum was a region in the blue between 466 and 478 $\mu\mu$. In this region an exposure of only 4 seconds sufficed to call forth heliotropic curvatures in 50 percent of the stems of the seedlings. For longer waves a longer exposure was required, thus for a wave length of 499 $\mu\mu$ an exposure of 120 seconds, and for a wave length of 534 $\mu\mu$ an exposure of 6300 seconds. The yellow and red parts of the spectrum seemed absolutely ineffective.

We undertook last summer a series of experiments to ascertain the relative efficiency of the various parts of a carbon arc spectrum on the newly regenerated polyps of *Eudendrium*. The spectrum was thrown on a glass trough with parallel walls which contained a row of *Eudendrium* stems with a number of newly regenerated polyps. The position of each polyp was noted at the beginning of the experiment. Loeb and Ewald had found that the minimal time of exposure to produce heliotropic curvatures in more than 50 percent of the polyps is, for the same intensity of light, considerably greater than in the case of the seedlings of *Avena*. It required an exposure of at least 5 minutes in order to call forth heliotropic curvatures of the polyps of *Eudendrium* in the spectrum. The experiments were rendered difficult by the fact that the young polyps are very delicate and suffer easily. If a stem with sickly polyps is included in an experiment it is liable to disturb the

result. It was found that for the polyps of *Eudendrium* the most efficient region of the spectrum was in the blue in the neighborhood of 4735 angstrom units, which coincides with the region of maximal efficiency found for the seedlings of oats by Blaauw; while the region from 4900 to 5300 and from 4690 to 4400 or below was still effective, but less than the region in the blue near 4735. The red and yellow rays were practically ineffective, at least an exposure of over 5 hours to the red and yellow rays induced no curvature. The following table may serve to illustrate this statement.

The fact that in the first series (5-minute exposure) one polyp bent to the light in the red was, as the other experiments showed, merely

TIME	WAVE-LENGTH	COLOR	POLYPS BENT TO THE LIGHT		TIME	WAVE-LENGTH	COLOR	POLYPS BENT TO THE LIGHT	
	Angstroms		Fraction	Per cent		Angstroms		Fraction	Per cent
5 min.	6500 A	red	1/29	4	15 min.	6700–5400	red to yellowish green	0	0
	6000	yellow	0/4	0		5400–4900	yellowish-green and green	14/37	38
	5700	yellow	0/13	0		4900–4100	blue and violet	72/95	76
	5300–5345	yellowish-green	5/15	33		4100–3700	violet	14/30	46
	5100	green	3/12	25	5½ hrs.	6560–6000	red and orange	0/32	0
	4900	blue	11/32	35		5720	yellow	0/21	0
	4735	blue	30/49	62		5500–5300	green	3/13	24
	4690	blue	4/21	19		4950	blue	1/6	16
	4600	blue	5/22	23		4800	blue	7/16	44
	4400	blue	5/52	10		4700	blue	24/24	100

an accident. The region of maximal efficiency was in the blue of about 4735 A.

In the second series (15-minute exposure) no bending occurred in the red and yellow, in the green and bluish-green less than 50 percent bent to the light, in the blue 76 percent, and in the violet less than 50 percent.

The final series (5½-hour exposure) is given to illustrate the fact that even an exposure of 5½ hours does not induce any bending in the red and yellow, while in the blue in the region of 4700 all the polyps were bent to the light. It should be mentioned that this effect was already reached in the region of 4700 A. in 160 minutes; after an exposure of 10 minutes already 73 percent were bent to the light.

These experiments show that the relative efficiency of the different parts of the spectrum of a carbon arc light for the production of heliotropic curvatures in the animal *Eudendrium* and in the seedlings of the plant *Avena* is practically identical.

[1] Blaauw, *Rec. des Trav. botaniques Néerlandais*, 5, 209 (1909).
[2] Loeb and Ewald, *Zentralbl. f. Physiol.*, 27, 1165 (1914).

ARCHAEOLOGY OF BARBADOS

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Our knowledge of the extent, character, and relationship of the prehistoric population of Barbados is very indefinite. It is not known whether the island was inhabited when discovered by the Portuguese in 1505, but there is every reason to believe it was, for a few aborigines still remained when it was colonized by the English over a century later. The existence of a native population is shown by Lignon's map published in 1657, thirty-one years after the English landed at Holetown, on which we find legends referring to Indian settlements. Several writers assert that while a limited number of Caribs were found by the first English colonists, these should be regarded as transient visitors from neighboring islands, rather than permanent occupants.

Archaeological evidences of a considerable prehistoric population in Barbados before the advent of Europeans are somewhat more definite than historical. These have already been presented by Greville T. Chester, and other archaeologists who have described many shell celts collected on Barbados. They have also brought to the attention of students numerous village sites that show evidences of a long continued occupation.

In a brief reconnaissance* made on Barbados by me in 1902, an examination was made of the archaeological evidence and data gathered bearing on the age and nature of the culture it revealed. An attempt is here made to determine the relation of this material to that found on the other West Indies. I am convinced from this examination that Barbados had a large local population in prehistoric times,

*These studies were made under the auspices of the Heye Museum of New York and the Bureau of American Ethnology of the Smithsonian Institution; a more complete account, amply illustrated, will be published later by the Bureau.

and that its culture, with certain significant differences, resembled in general that on the other West Indies.

The presence of an aboriginal population in Barbados is indicated by the numerous prehistoric objects collected in caves, natural and artificial, mounds, middens, and other village sites. The number of prehistoric implements found in these localities on Barbados, is so large that there can be no doubt that the island once had a great aboriginal population. These objects were made of shell, and were formerly so abundant that, according to several informants, they were ground up and used for road beds. Even now a considerable number can be secured from the natives at a few pennies apiece, for the asking.

There are several well-marked types of shell implements, so characteristic that there is no doubt they are of local manufacture. Stone celts are very rare for the reason that suitable material for their manufacture does not exist in any place on the island. The few that occur were evidently brought to Barbados from over the sea. This abundance of shell implements and relative paucity of those made of stone has an important bearing on whether the aborigines were transient visitors or permanent inhabitants. They must have been locally made by the latter, for shell implements are rare on the nearest islands where stone implements are very abundant.

Evidences that the aborigines of Barbados formerly inhabited natural caves are many. On the roofs of these caves there still remain marks of smoke indicating former fires, while in their floors, especially near the entrance, numerous artifacts have been excavated. Joseph Forte found over a hundred implements in one cavern, 350 feet above the level of the sea. Although the material out of which they were made is different, the forms of these objects show that the culture of the cave dwellers resembled that of cave people found elsewhere in the West Indies, although much more varied.

While there is nothing in the natural caves and their artificial contents which shows that the aboriginal culture of Barbados was exceptional, the claim that artificial caves exist is important; for if it can be successfully proved that the aborigines made these caves, Barbados is the only West Indian island where artificial cave dwellings were made. The existence of this kind of cave from its unique character is naturally doubted by many, and while some writers deny their artificial origin, or ascribe them to Europeans, others take the opposite view. The examinations made by me lead me with some hesitation to adopt the latter conclusion.

Three groups of artificial caves, supposed to have been the work of historic aborigines, were seen by me on my visit to Barbados. One of these, and the most problematical, has been known for several centuries from its site as Indian Castle; the others are pit-like excavations near Bridgetown on the right bank of Indian River, and at Freshwater Bay.

The so-called Indian Castle is situated northeast of Speightown, on the Pleasant Hill Estate, about three miles due east of Six Mens Bay. As one leaves the town above mentioned the road rises gradually to a hill and passes the 'Castle,' revealing the entrance to the cave on the right-hand side in the cliff above the road. The cave opens through the *archway with a key-stone* on which is cut a rude figure in relief. Both entrances and arch have smooth perpendicular walls with well made angles in each corner, the whole chamber cut out of the solid rock. The subterranean room has lateral recesses in its walls and smaller mural niches. An opening in the right-hand wall communicates with a well, also excavated in rock, but open above to the sky.

The whole appearance of this excavation, especially the conical apex of the second chamber, led me at my first visit to regard it a lime-kiln made by Europeans, but it greatly differs from one of these pits in several particulars. This cave has been known for many years. The Reverend Griffith Hughes speaks of this place in 1750, or about 125 years after the island was colonized, as follows: "A very commodious one [cave] in the side of a neighboring hill called to this day Indian Castle and almost in a direct line from Six Mens Bay, ..." "Among several broken fragments of idols said to be dug up in this place," continues the Reverend Hughes, "I saw the head of one which alone weighed 60 pds. weight." An adjacent pond, he writes, "since the memory of oldest inhabitants has been called Indian Pond." We have in fact circumstantial evidence that the cave is aboriginal, and yet not ample data to prove that the cave was excavated by Indians in prehistoric times.

There is, however, more trustworthy evidence that the other excavated chambers in the Barbados ascribed to prehistoric man were made by the aborigines. Unlike the Indian Castle cave and natural caves above mentioned, the 'Indian excavations' are dug into the surface rock like pit dwellings. They lie near undoubted Indian middens or sites of aboriginal settlements which tells in favor of their aboriginal character. Like the Castle cave they also are ascribed by some authors to early colonists, but the majority call them Indian excavations and Carib graves. It is instructive to note that abundant evidences of

aboriginal occupation occur in their immediate vicinity, and there appears no good reason to question the theory that their local name, Indian excavations, correctly ascribes them to Indians.

Mounds or middens of considerable size, indicating aboriginal settlement, occur at many points on Barbados, especially the lee shore. The majority are situated on the plains skirting the coast, but many are found inland, especially where the soil invites the agriculturalist. Among these may be mentioned: (1) Near St. Luke's Chapel; (2) Indian River; (3) Freshwater Bay; (4) Codeington Estate Springs; (5) Maxwells; (6) New South Point Lighthouse; (7) Three Houses; (8) Marl Hill; (9) Speightown; (10) Holetown, and many other places. On account of long cultivation of the fields the size and appearance of many aboriginal sites have been more or less obscured, although their former existence can still be discovered.

A few undescribed specimens of aboriginal pottery found in these places are valuable in determining the culture of those who inhabited these localities. One of the most perfect of these having a graceful form like the well known Arawak "monkey vase," was found by Mr. Taylor at Indiantown near Bridgetown. This object can hardly be distinguished from Arawak pottery found in Grenada and St. Kitts. Another large flat fragment of pottery, from the same locality has the form of a griddle, resembling those on which the Arawaks bake their cassava cakes. A clay idol resembling one found in Porto Rico is suggestive of similar forms of idolatry in the two islands.

The general characters of aboriginal life in Barbados as revealed by these and many other archaeological remains, show that the culture was somewhat different from that of Porto Rico, the neighboring Lesser Antilles, or the coast of South America, but that the prehistoric people belonged to the same Carib-Arawak stock that inhabited these localities in prehistoric times. The number and variety of the artifacts show that the prehistoric population of Barbados was large and implies that it had diminished to such an extent that when England began to colonize the island, only a few survivors remained. These prehistoric inhabitants of the Barbados were practically cave dwellers and although they had not reached the stage of culture characteristic of Porto Rico and Hayti, they were probably the only Antilleans who made artificial caverns for habitations or other purposes, and carried the manufacture of shell implements to the highest degree of perfection. Their culture was not the most advanced among the Antilleans, but it was characteristic, showing in a marked manner the influence of their geographical environment. Although there are minor differences in artifacts, indi-

cating well marked sub-areas—of which Barbados is one—in the different West Indies there is a certain unity which stamps them as belonging to one great culture area—the Carib-Arawak—extending through the islands from the heart of South America to Florida. This resemblance extending along the east coast of Central America to the Maya area, with which it has close affinities, and along the north coast of South America to the mouth of the Amazon, is practically identical with the Arawak.

CULTURE OF THE NORTH AMERICAN INDIANS OCCUPYING THE CARIBOU AREA AND ITS RELATION TO OTHER TYPES OF CULTURE

By Clark Wissler

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK

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The anthropology of North America has now reached a stage in its development where larger and broader problems can be successfully pursued. In the past, for want of data and tried methods, investigations were of necessity confined to tribal units and it was not until a considerable number of these units had been studied that any positive conclusions could be formed as to the continent as a whole. The following brief statement is a mere summary of investigations bearing upon the origin and significance of the observed distribution of certain culture traits in the northern half of the continent. The method is to study in detail the collections in museums and to correlate the results with the field-data of anthropologists and the observations of travelers.

A brief general discussion of the problem may be found in the review of the 'Material Culture of the North American Indians' in the *American Anthropologist*, vol. 16, no. 2, and in special studies, as 'The Influence of the Horse in the Development of Plains Culture,' *American Anthropologist*, vol. 16, no. 1, and the 'Material Culture of the Blackfoot,' *Anthropological Papers*, American Museum of Natural History, vol. 5, part 1. These, however, are but preliminary to the more exhaustive treatment of the problems now under investigation.

1. *Caribou Culture Defined.* The material cultures of the Indian tribes occupying the interior of Canada present striking similarities. Their chief food was animal, the flesh of the caribou. Excepting the Pacific drainage and the prairie section, the entire area of Canada, including the interior of Alaska and the Arctic islands of the north, constituted the range of the caribou. Their range and that of the

bison overlapped in the Saskatchewan and Mackenzie Basins. Throughout this extensive territory the Indians were more or less completely dependent upon the caribou for food and clothing. Even the Eskimo hunted them in summer and made their winter clothing of their skins. Naturally, the material life of the Indians, and to a large extent that of the Eskimo, was adjusted to the necessities of a caribou hunter's career. For this reason, we speak of the culture of these Indians as the caribou culture. Some of the traits of this culture are: the use of birch and other bark for canoes, vessels, etc., the toboggan and the snowshoes, dog traction, the general use of snares for taking game both large and small, the high development of the net, the soft moccasin and the hooded coat, special methods of dressing and smoking skins, and the more or less exclusive use of a tipi-like shelter.

2. *The Relation of Caribou Culture to Bison Culture.* The Indian tribes formerly occupying the plains and prairies in the interior of North America constitute a highly individualized cultural group. They were almost entirely dependent upon the bison. As we have just noted, the bison and caribou areas overlapped, the ranging habits of the two animals have certain similarities; thus they move in great herds, and in the open country, and may be captured en masse by being driven into pounds or other enclosures. Our main problem is the analytic comparison of these two cultures with a view to determining their possible historical connection.

(*a*) *Bison and Caribou Drives.* From Alaska to Greenland the methods of driving herds of caribou in pens, snares, or between the lines of hunters are in general the same. The last method is rarely used in winter, while the former is almost exclusively a winter method. Long lines of poles are set up in the snow to drive the running caribou, which show a tendency to run along such lines, but men and women may be placed behind screens to turn the leaders in case of need. By long converging lines a herd may be run over a bank into an enclosure, into snares, or on to thin ice. In a precisely similar manner the Indians of the plains impounded buffalo in winter and occasionally in summer. From the detailed accounts, it is clear that even many minutiae of manipulation are the same for the bison and caribou. Another point is that the farther north one goes among the plains Indians the more highly developed was the method of impounding, and according to early literature the Cree were the most skillful. Now it so happens that the Cree living in the forests and tundras are also skillful in driving the caribou, giving us the probable connecting link.

(*b*) *Dog Traction.* Though the dog was quite generally distributed in America it was chiefly in the bison and caribou areas that he was used for transport. The use of sledges was practically confined to the caribou area but in summer the loads were made up as packs. In the bison area and the Barren Grounds tent poles were dragged by dogs, and in parts of the former area developed into a packing drag known as a travois. In this connection we have investigated the relation of the horse to bison culture making it clear that one of the reasons why the use of the horse spread so rapidly in this area was that he could be quickly substituted for the dog without developing new culture traits. Our data show that the horse quickly reached the tribes of Canada, via the Spanish settlements of Mexico, and was carried as far north as the climate permitted.

(*c*) *The Tipi-like Shelter.* The development of this problem has been the detailed study of the structure of tipis and tents of the two areas. The most intense development of the true tipi is found in the bison area, the Mackenzie Valley, and among the Algonkin speaking tribe of Canada. The definitive points of comparison are the arrangements of the poles forming the conical foundation and the detailed cut of the skin or bark covers. An analytic comparison of structure along these lines suggests that the tipi was introduced into the bison area from the caribou area to the north.

(*d*) *Costume.* Another important part of the investigation is the study of museum collections of costume, the related techniques of sewing and skin dressing. In this case we have a contemporary, Gudmund Hatt, a young Danish anthropologist who has made an intensive study of Lapp and Siberian costumes and pointed out some correspondences in America. Our own data for the bison area in contrast to that for the eastern and western parts of the United States make it clear that we have in this area alone a development of skin clothing comparable to that of the caribou area. Further, the detailed study of forms in both men's and women's clothing, shows that in general, as we proceed northward in the bison area, the resemblances to the culture of the caribou area become more numerous. In brief, the conclusion is that in America, the use of skin clothing is practically confined to the combined bison and caribou areas but is a more intense and universal trait in the latter. We find a similar distribution for the true moccasin and certain methods of dressing skins.

(*e*) *Harpoons and Hunting Points.* The investigation of these traits of culture has not gone far enough to make very definite statements. One characteristic of the caribou area is the large use of bone points,

many of which are barbed. While barbed points are rarely used in the bison area, there was a strong tendency to use bone points, especially in the north.

In addition to the preceding, the investigation is concerned with such traits as textile arts, birchbark technique, weapons, and special manufacturing processes. The distribution of the traits enumerated above indicates a fundamental similarity between the material cultures of the caribou and bison areas. The interpretation of this observation is an important theoretical problem. The experience of anthropologists to date is that in all such cases we have two major alternatives, diffusion from a single center or independent development in two or more localities. It remains to be seen which of these will be the more satisfactory interpretation for the above noted similarities in culture.

Finally, this study has developed the problem of caribou, or New World reindeer, culture in its relation to the reindeer culture of the Old World in both modern and paleolithic times. It now appears probable that in the great area of the reindeer and caribou (for they are geographically continuous) we have a concomitant human culture which may be as old as paleolithic man in Europe. This will be more fully discussed when our detailed studies of caribou culture are published.

REPORT ON THE AUTUMN MEETING

Prepared by the Home Secretary

The Autumn Meeting of the National Academy of Sciences was held in the Botanical Laboratory of the University of Chicago on December 7, 8, and 9, 1914, twenty-one members of the Academy being present.

BUSINESS SESSIONS

Business sessions were held on December 8 and 9, at which the following business was transacted:

The President announced that since its Annual Meeting in April the Academy had lost by death two members, Theodore Nicholas Gill, elected in 1873, who died on September 25, 1914, and Charles Sedgwick Minot, elected in 1897, who died on November 19, 1914; also two foreign associates, Edouard Suess, elected in 1898, who died on April 26, 1914, and August Weismann, elected in 1913, who died on November 5, 1914.

The President made also the following announcements:

That Mr. Ira Remsen was acting as Chairman of the Board of Directors of the Bache Fund for the time being in the place of Charles S. Minot, deceased.

That by the death of Theodore N. Gill a vacancy was created in the Finance

Committee, and that this had been filled by the appointment of Mr. Arnold Hague.

That the following standing committees of the Academy had been chosen by the Council in accordance with article 4, section 4, of the Constitution:

COMMITTEE ON NOMINATIONS

	Noyes, A. A. (Chairman)	
Chittenden, R. H.	Hague, Arnold	Trelease, William
Comstock, G. C.	Holmes, W. H.	Woodward, R. S.
Conklin, E. G.	Moore, E. H.	

COMMITTEE ON MATHEMATICS

	Moore, E. H. (Chairman)	
Bocher, Maxime	Moulton, F. R.	Van Vleck, E. B.
Bolza, Oskar	Osgood, W. F.	Webster, A. G.
Dickson, L. E.	Story, W. E.	Woodward, R. S.

COMMITTEE ON ASTRONOMY

	Comstock, G. C. (Chairman)	
Barnard, E. E.	Frost, E. B.	Moulton, F. R.
Campbell, W. W.	Hale, G. E.	Pickering, E. C.
Elkin, W. L.	Leuschner, A. O.	Woodward, R. S.

The President also announced that he had appointed the following committees:

COMMITTEE ON THE PROGRAM FOR THE MEETINGS

	Boltwood, B. B. (Chairman)	
Cattell, J. McK.	Mall, F. P.	Frost, E. B.

AUDITING COMMITTEE

Clarke, F. W.	Dall, W. H. (Chairman)	Day, A. L.

COMMITTEE ON THE REVISION OF THE CONSTITUTIONAL RULES

Day, A. L.	Cross, Whitman (Chairman)	Walcott, C. D.

The President further announced that committees on trust funds had been reappointed in accordance with Rule 27 as follows, the numbers following the names showing the date at which the term of service expires.

THE HENRY DRAPER FUND

	Hale, G. E. (Chairman), 1918	
Wright, A. W., 1915	Trowbridge, John, 1916	Campbell, W. W., 1919
	Michelson, A. A., 1917	

THE J. LAWRENCE SMITH FUND

	Morley, E. W. (Chairman), 1919	
Pumpelly, R., 1915	Remsen, Ira, 1916	Dana, E. S., 1918
	Hague, Arnold, 1917	

THE COMSTOCK FUND

	NICHOLS, E. L. (Chairman), 1918	
THOMSON, ELIHU, 1915	CREW, HENRY, 1916	NOYES, A. A., 1919
	WEBSTER, A. G., 1917	

THE MURRAY FUND

DALL, W. H., 1915	HAGUE, ARNOLD, 1916	PARKER, G. H., 191

The Committee on the Amendment to the Act of Incorporation of the Academy reported through Mr. Charles D. Walcott, its Chairman, that the following amendment had been passed by both houses of Congress during the last session and approved by the President on May 27, 1914:

(Public—No. 109—63d Congress. S. 4096)

An Act to amend the Act authorizing the National Academy of Sciences to receive and hold trust funds for the promotion of science, and for other purposes.

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That the Act to authorize the National Academy of Sciences to receive and hold trust funds for the promotion of science, and for other purposes, approved June twentieth, eighteen hundred and eighty-four, be, and the same is hereby, amended to read as follows:

"That the National Academy of Sciences, incorporated by the Act of Congress approved March third, eighteen hundred and sixty-three, be, and the same is hereby, authorized and empowered to receive, by devise, bequest, donation or otherwise, either real or personal property, and to hold the same absolutely or in trust, and to invest, reinvest, and manage the same in accordance with the provisions of its constitution, and to apply said property and the income arising therefrom to the objects of its creation and according to the instructions of the donors: Provided, however, That the Congress may at any time limit the amount of real estate which may be acquired and the length of time the same may be held by said National Academy of Sciences."

Sec. 2. That the right to alter, amend, or repeal this Act is hereby expressly reserved.

Approved, May 27, 1914.

At the business session on December 8, 1914 the Committee on the Revision of the Constitution and Rules reported a series of amendments to the constitution of the Academy, which, after discussion, were referred to the Council for consideration and report. The Council presented its report, with further amendments, to the Academy at the stated session on December 9, 1914; and the amended report was adopted by the Academy, sitting as a committee of the whole. This report will come up for final consideration by the Academy at the stated meeting in April 1915.

A new Rule was adopted, reading as follows: The holders of the Medal for Eminence in the Application of Science to the Public Welfare shall be notified, like members, of the meetings of the Academy, and invited to participate in its scientific sessions.

Rule III was amended, by replacing the second and third sentences, so as to read as follows: That it shall be the duty of the Auditing Committee to verify the records, receipts, and disbursements maintained by the Treasurer and the agreement of book and bank balances; to examine all securities in

the custody of the Treasurer, and to compare the stated income of such securities with the receipts of record; to examine all vouchers covering disbursements for the account of the Academy and the authority therefor, and to compare them with the Treasurer's record for expenditures; to examine and verify the account of the Academy. The auditing committee may employ the services of an expert accountant to assist in the examination of the books of the Treasurer.

The recommendation of the Committee on the Henry Draper Fund to award the Draper medal in 1915 to Prof. Joel Stebbins, of the University of Illinois, in recognition of his work on Application of the Selenium Cell to Stellar Photometry, was approved.

A resolution of the Council providing for a special business meeting and a special scientific meeting of the Academy on April 19, 1915, to be followed by the stated meeting on Tuesday and Wednesday, April 20 and 21, 1914, as required by the constitution, was unanimously adopted.

It was voted that the thanks of the National Academy of Sciences be given to the Chairman, Mr. E. H. Moore, and to the members of the local committee for their most successful arrangements for the Autumn Meeting held in Chicago, December 7, 8, 9, 1914.

It was further voted that the thanks of the Academy be extended to President Judson, to the University of Chicago, to the Quadrangle Club and to the Chaos Club for their aid in arranging the social features which contributed so much to the enjoyment of the Autumn Meeting.

SCIENTIFIC SESSIONS

Two public lectures on the WILLIAM ELLERY HALE FOUNDATION were given on December 7 and 8 by WILLIAM WALLACE CAMPBELL, Director of the Lick Observatory, on Stellar Evolution and the Formation of the Earth.

Four public scientific sessions were held on December 7, 8, and 9, at which the following papers were presented:

1. W. W. CAMPBELL: On the radial velocities of nebulae.
2. HEBER D. CURTIS (introduced by W. W. Campbell): Preliminary note on nebular proper motions.
3. R. A. MILLIKAN (introduced by A. A. Michelson): The coefficient of slip in gases and its relation to the nature of the impact between a molecule of a gas and the surface of a solid or liquid.
4. W. D. HARKINS and E. C. HUMPHREY (introduced by Julius Stieglitz): The capillary and electrical forces at the interface between two liquids.
5. HERBERT N. MCCOY (introduced by Julius Stieglitz): The solubilities of radium compounds as indicated by the solubilities of analogous compounds of calcium, strontium, and barium.
6. L. A. BAUER (introduced by George E. Hale): Present status of the general magnetic survey of the globe.
7. CHARLES J. CHAMBERLAIN (introduced by J. M. Coulter): A phylogenetic study of cycads.
8. C. M. CHILD (introduced by J. M. Coulter): A dynamic conception of the organic individual.

9. S. W. WILLISTON (introduced by T. C. Chamberlin): The American land vertebrate fauna and its relations.

10. A. J. CARLSON (introduced by J. M. Coulter): Some new observations on the physiology of the stomach in man.

11. LEONARD E. DICKSON: Recent progress in the theories of modular and formal invariants.

12. HENRY S. WHITE (introduced by L. E. Dickson): The synthesis of triad systems Δ_t in t elements, in particular for $t = 31$.

13. E. J. WILCZYNSKI (introduced by E. H. Moore): Conjugate systems of space curves with equal Laplace-Darboux invariants.

14. EDWIN B. FROST: An interesting stellar system.

15. GEORGE E. HALE: The direction of rotation of solar storms.

16. A. A. MICHELSON: Behavior of metals and other substances near the rupture point.

17. C. W. BALKE and GEORGE W. SEARS (introduced by W. A. Noyes): The atomic weight of tantalum.

18. E. W. WASHBURN (introduced by W. A. Noyes): Our systematic knowledge of the properties and behavior of solutions of non-electrolytes.

19. T. C. CHAMBERLIN: The fundamental segmentation of the earth.

20. WILLIAM TRELEASE: Phoradendron.

21. CHARLES E. ALLEN (introduced by E. B. Van Vleck): Development of the male germ cells of polytrichum.

22. C. T. KNIPP (introduced by W. A. Noyes): Experimental data on the stability of positive and negative ions.

23. S. W. PARR (introduced by W. A. Noyes): The development of an acid-resisting alloy for a bomb calorimeter.

24. W. L. TOWER (introduced by J. M. Coulter): Experimental production of a new ordinal character.

25. G. A. MILLER (introduced by L. E. Dickson): The ϕ-subgroup of a group of finite order.

26. E. E. BARNARD: Explanation of certain phenomena of the tail of comet Morehouse (III, 1908).

27. PHILIP FOX (introduced by E. B. Frost): The rotation-period of the sun.

28. J. C. KAPTEYN and W. S. ADAMS: On the relations between the proper motions and the radial velocities of the stars of the spectral types F, G, K, and M.

29. S. B. NICHOLSON (introduced by W. W. Campbell and A. O. Leuschner): Discovery of a ninth satellite of Jupiter.

30. FRANK R. LILLIE (introduced by J. M. Coulter): The fertilizing power of sperm dilutions.

31. GILBERT AMES BLISS (introduced by E. H. Moore): A generalization of a theorem of Gauss concerning geodesic triangles.

32. F. R. MOULTON: An extension of the process of successive approximations for the solution of differential equations.

33. JULIUS STIEGLITZ: Molecular rearrangements of triphenylmethyl derivatives.

34. WILLIAM CROCKER and J. F. GROVES (introduced by J. M. Coulter): Methods of determining the life-duration of seeds.

35. EDWIN O. JORDAN (introduced by J. M. Coulter): Variation in bacteria.

36. SHIRO TASHIRO (introduced by J. M. Coulter): On the nature of nerve impulse.

37. ELIAKIM H. MOORE: On the integration by successive approximations of the ordinary differential equation of the first order in general analysis.

38. W. H. WRIGHT (introduced by W. W. Campbell): On the occurrence of the line 4886A and the related series of lines in the spectra of the planetary nebulae.